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I. INTRODUCTION AND FOCUS QUESTIONS

Have you ever wondered about how to identify the side lengths of a square box or the dimensions of a square lot if you know its area? Have you tried solving for the length of any side of a right triangle? Has it come to your mind how you can find the radius of a cylindrical water tank?

Find out the answers to these questions and understand the various applications of radicals to real-life situations.
II. LESSONS AND COVERAGE

In this module, you will examine the questions on page 225 as you take the following lessons.

*Lesson 1 – Zero, Negative Integral, and Rational Exponents*
*Lesson 2 – Operations on Radicals*
*Lesson 3 – Application of Radicals*

**Objectives**

In these lessons, you will learn to:

| Lesson 1 | • apply the laws involving positive integral exponents to zero and negative integral exponents.  
|          | • illustrate expressions with rational exponents.  
|          | • simplify expressions with rational exponents.  |
| Lesson 2 | • write expressions with rational exponents as radicals and vice versa.  
|          | • derive the laws of radicals.  
|          | • simplify radical expressions using the laws of radicals.  
|          | • perform operations on radical expressions.  |
| Lesson 3 | • solve equations involving radical expressions.  
|          | • solve problems involving radicals.  |
Module Map

Here is a simple map of the lessons that will be covered in this module.
III. PRE-ASSESSMENT

Part I

Find out how much you already know about this module. Choose the letter that you think best answers the questions. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. What is the simplified form of \(4^{5} \cdot 6^{-1} \cdot 100^{-1} \cdot \frac{1}{2^{5}}\)?
   
   a. 1 
   b. \(\frac{1}{75}\) 
   c. \(\frac{1}{150}\) 
   d. \(\frac{1}{6000}\)

2. Which of the following is true?
   
   a. \(5^{1} + 5^{3} = 5^{5}\) 
   b. \(\frac{2^{6}}{2^{3}} = 2^{3}\) 
   c. \(\left(\frac{1}{3}\right)^{2} = 9^{\frac{2}{3}}\) 
   d. \(4^{\frac{2}{3}} = \frac{1}{4^{\frac{5}{3}}}\)

3. What is the equivalent of \(\sqrt{4} + \sqrt{2}\) using exponential notation?
   
   a. \(4^{\frac{1}{3}} + 2^{\frac{1}{3}}\) 
   b. \(4^{\frac{1}{3}} + 2^{\frac{1}{3}}\) 
   c. \(6^{\frac{1}{3}}\) 
   d. \(6^{\frac{1}{3}}\)

4. Which of the following radical equations will have \(x = 6\) as the solution?
   
   a. \(\sqrt{x} - 2x + 7 = 0\) 
   b. \(2\sqrt{x} - 3 = x - 3\) 
   c. \(\sqrt{x} = 9\) 
   d. \(3\sqrt{x} = 5\)

5. What is the result after simplifying \(2\sqrt{3} + 4\sqrt{3} - 5\sqrt{3}\)?
   
   a. \(-\sqrt{3}\) 
   b. \(\sqrt{3}\) 
   c. \(11\sqrt{3}\) 
   d. \(21\sqrt{3}\)

6. Which of the following is the result when we simplify \((2\sqrt{8} + 3\sqrt{5})(6\sqrt{8} + 7\sqrt{5})\)?
   
   a. \(12\sqrt{64} + 14\sqrt{40} + 18\sqrt{40} + 21\sqrt{25}\) 
   b. \(12\sqrt{8} + 32\sqrt{40} + 21\sqrt{5}\) 
   c. \(201 + 64\sqrt{10}\) 
   d. \(195\sqrt{10}\)

7. What is the result when we simplify \(\frac{6 - \sqrt{2}}{4 - 3\sqrt{2}}\)?
   
   a. 5 
   b. \(-2\sqrt{2}\) 
   c. \(5 - \sqrt{2}\) 
   d. \(-9 - 7\sqrt{2}\)
8. What is the simplified form of $\frac{\sqrt{3}}{\sqrt{3}}$?
   a. $\sqrt{3}$  
   b. $\frac{\sqrt{3}}{3}$  
   c. $\sqrt{27}$  
   d. $\frac{\sqrt{27}}{3}$

9. Luis walks 5 kilometers due east and 8 kilometers due north. How far is he from the starting point?
   a. $\sqrt{89}$ kilometers  
   b. $64$ kilometers  
   c. $\sqrt{39}$ kilometers  
   d. $\sqrt{25}$ kilometers

10. Find the length of an edge of the given cube.
   [Surface Area = 72 sq. m]
   a. $6\sqrt{2}$ meters  
   b. $6\sqrt{12}$ meters  
   c. $2\sqrt{3}$ meters  
   d. $\sqrt{2}$ meters

11. A newborn baby chicken weighs $3^2$ pounds. If an adult chicken can weigh up to 34 times more than a newborn chicken. How much does an adult chicken weigh?
   a. 9 pounds  
   b. 10 pounds  
   c. 64 pounds  
   d. $\frac{144}{9}$ pounds

12. A giant swing completes a period in about 15 seconds. Approximately how long is the pendulum’s arm using the formula $t = 2\pi \sqrt{\frac{l}{32}}$, where $l$ is the length of the pendulum in feet and $t$ is the amount of time? (use: $\pi \approx 3.14$)
   a. 573.25 feet  
   b. 182.56 feet  
   c. 16.65 feet  
   d. 4.31 feet

13. A taut rope starting from the top of a flag pole and tied to the ground is 15 meters long. If the pole is 7 meters high, how far is the rope from the base of the flag pole?
   a. 2.83 meters  
   b. 4.69 meters  
   c. 13.27 meters  
   d. 16.55 meters

14. The volume ($V$) of a cylinder is represented by $V = \pi r^2 h$, where $r$ is the radius of the base and $h$ is the height of the cylinder. If the volume of a cylinder is 120 cubic meters and the height is 5 meters, what is the radius of the base?
   a. 2.76 meters  
   b. 8.68 meters  
   c. 13.82 meters  
   d. 43.41 meters
Part II: (for nos. 15-20)

Formulate and solve a problem based on the given situation below. Your output shall be evaluated according to the given rubric below.

You are an architect in a well-known establishment. You were tasked by the CEO to give a proposal for the diameter of the establishment’s water tank design. The tank should hold a minimum of 800 cm³. You were required to present a proposal to the Board. The Board would like to see the concept used, its practicality and accuracy of computation.

<table>
<thead>
<tr>
<th>Categories</th>
<th>2 Satisfactory</th>
<th>1 Developing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Concept</td>
<td>Demonstrate a satisfactory understanding of the concept and use it to simplify the problem.</td>
<td>Demonstrate incomplete understanding and have some misconceptions.</td>
</tr>
<tr>
<td>Accuracy of Computation</td>
<td>The computations are correct.</td>
<td>Generally, most of the computations are not correct.</td>
</tr>
<tr>
<td>Practicality</td>
<td>The output is suited to the needs of the client and can be executed easily.</td>
<td>The output is suited to the needs of the client but cannot be executed easily.</td>
</tr>
</tbody>
</table>

IV. LEARNING GOALS AND TARGETS

After going through this module, you should be able to demonstrate an understanding of key concepts of rational exponents, radicals, formulate real-life problems involving these concepts, and solve these with utmost accuracy using a variety of strategies.
What to KNOW

Start Lesson 1 of this module by assessing your knowledge of laws of exponents. These knowledge and skills may help you understand zero, negative integral, and rational exponents. As you go through this lesson, think of the following important question: “How do we simplify expressions with zero, negative integral, and rational exponents? How can we apply what we learn in solving real-life problems?” To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier.

➤ Activity 1: Remember Me this Way!

A. Simplify the following expressions.

1. $b^5 \cdot b^3$
2. $\left( \frac{r^2}{s^3} \right)^3$
3. $(-2)^3$
4. $\frac{10m^6}{2m^{10}}$
5. $(m^3)^5$

B. Solve the given problem then answer the questions that follow.

The speed of light is approximately $3 \times 10^8$ meters per second. If it takes $5 \times 10^2$ seconds for light to travel from the sun to the earth, what is the distance between the sun and the earth?

Questions:
1. How did you solve the given problem?
2. What concepts have you applied?
3. How did you apply your knowledge of the laws of integral exponents in answering the given problem?

The previous activity helped you recall the laws of exponents which are necessary to succeed in this module.

Review: If $a$ and $b$ are real numbers and $m$ and $n$ are positive integers, then

- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left( \frac{a}{b} \right)^m = \frac{a^m}{b^m}$, $b \neq 0$
- $a^m \cdot a^n = a^{m-n}$, if $m > n$, $a \neq 0$
- $\frac{a^m}{a^n} = a^{m-n}$, if $m < n$, $a \neq 0$
- $\frac{1}{a^n} = a^{-n}$, if $m < n$, $a \neq 0$

In the next activity, your prior knowledge on zero, negative integral, and rational exponents will be elicited.
**Activity 2: Agree or Disagree!**

Read each statement under the column STATEMENT then write A if you agree with the statement; otherwise, write D. Write your answer on the “Response-Before-the-Discussion” column.

<table>
<thead>
<tr>
<th>Response-Before-the-Discussion</th>
<th>Statement</th>
<th>Response After the Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Any number raised to zero is equal to one (1).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>An expression with a negative exponent CANNOT be written as an expression with a positive exponent.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2^{-3}$ is equal to $\frac{1}{8}$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Laws of exponents may be used to simplify expressions with rational exponents.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left(\frac{1}{3}\right)^{-2} = 9$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3^{0}4^{-2} = 16$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{(32x^{3}y^{4})^{-2}}$ may be written as $(32x^{3}y^{4})^{2}$ where $x \neq 0$ and $y \neq 0$</td>
<td>DO NOT ANSWER THIS PART YET!</td>
</tr>
<tr>
<td></td>
<td>$(-16)^{\frac{2}{3}} = -16$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The exponential expression $\frac{1}{x^{\frac{1}{2}} + 10^{\frac{1}{2}}}$ is equivalent to $\frac{1}{(x+10)^{\frac{1}{2}}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3^{2} \cdot 4^{0} + 1^{\frac{1}{2}} \cdot 5^{0} = 11$</td>
<td></td>
</tr>
</tbody>
</table>

You just tried to express your initial thoughts and ideas about our lesson. Let us answer the next activity that will deal with the application of negative integral exponents.
Activity 3: Play with the Negative!

Analyze the problem below then answer the questions that follow.

A grain of rice has a volume of $20^{-9}$ m³. A box full of rice has a volume of $20^{-3}$ m³. How many grains of rice are there in the box?

Questions:
1. What have you noticed from the values given in the problem?
2. What have you observed from the exponents?
3. What have you done to simplify these values?
4. How did you solve the problem?
5. Have you applied any law? Why?
6. Compare your answer with your classmates’ answers. What have you observed? Did you get the same answer? Why?

The previous activity introduced to you a real-life application of a negative exponent. Were you able to answer it correctly? Recall what you learned in Grade 7. If $a$ is a real number, $a \neq 0$, then $\frac{a^{-m}}{a^{-n}} = a^{m-n}$, if $m > n$. Remember this law of exponent as you do the next activity.

Activity 4: You Complete Me!

Fill in the missing parts of the solution in simplifying the given expression. Assume that $x \neq 0$, $a \neq 0$, and $h \neq 0$, then answer the process questions below.

1. \( \frac{h^5}{h^3} \rightarrow h^{?} \rightarrow h^? \rightarrow 1 \)
2. \( \frac{4^2}{4^2} \rightarrow 4^{? - ?} \rightarrow 4^? \rightarrow 1 \)
3. \( \frac{x^3}{x^5} \rightarrow x^{? - ?} \rightarrow x^? \rightarrow 1 \)
4. \( \frac{7^2}{7^2} \rightarrow 7^{? - ?} \rightarrow 7^? \rightarrow 1 \)
5. \( \frac{a^{12}}{a^{12}} \rightarrow ? \rightarrow ? \rightarrow ? \)

Questions:
1. What did you observe about the exponents?
2. How were the problems solved?
3. What can you conclude from the process of solving problems?
Let us now consolidate our results below.

**Definition of** \( a^0 \)

From Grade 7, we know that \( \frac{a^m}{a^n} = a^{m-n} \) if \( a \neq 0 \), \( m > n \). Suppose we want this law to hold even when \( m = n \).

Then \( \frac{a^m}{a^n} = a^{m-m} = a^0 \), \( a \neq 0 \).

But we also know that \( \frac{a}{a^n} = 1 \). Thus, we define \( a^0 = 1 \), \( a \neq 0 \).

Simplify the next set of expressions.

6. \( 3^{-2} \rightarrow \frac{1}{3^2} \rightarrow ? \rightarrow \frac{1}{9} \)

7. \( 4^{-2} \rightarrow \frac{1}{4^2} \rightarrow \frac{1}{16} \)

8. \( \left( \frac{1}{2} \right)^{-2} \rightarrow \frac{1}{\left( \frac{1}{2} \right)^2} \rightarrow \frac{1}{\frac{1}{4}} \rightarrow 2 \rightarrow 4 \)

9. \( \frac{1}{4^{-2}} \rightarrow \frac{1}{\frac{1}{4}} \rightarrow 1 \cdot \frac{4^2}{1} \rightarrow ? \)

10. \( \frac{1}{5^{-3}} \rightarrow ? \rightarrow ? \rightarrow ? \)

**Questions:**

1. What did you observe about the exponents?
2. How were the problems solved?
3. What can you conclude from the process of solving problems?

Let us now consolidate our results below.

**Definition of** \( a^{-n}, n > 0 \)

From Grade 7, we know that \( \frac{a^m}{a^n} = a^{m-n} \) if \( a \neq 0 \), \( m > n \). Suppose we want this law to hold even when \( m < n \).

Then \( \frac{a^m}{a^n} = a^{m-n} = a^n \), \( a \neq 0 \).

But we also know that \( \frac{a}{a^n} = \frac{1}{a} \). Thus, we define \( a^{-n} = \frac{1}{a^n}, a \neq 0 \).
In Grade 7 and in previous activities, you have encountered and simplified the following:

Positive Integral Exponent → \(3^2 = 9\) \(2^4 = 16\) \(a^3\)

Negative Integral Exponent → \(3^{-2} = \frac{1}{9}\) \(2^{-4} = \frac{1}{16}\) \(a^{-3} = \frac{1}{a^3}\)

Zero Exponent → \(3^0 = 1\) \(2^0 = 1\) \(a^0 = 1\)

Now, look at the expressions below.

\[
\begin{pmatrix}
\frac{1}{x^2} \\
\frac{1}{y^2} \\
\frac{1}{z^2} \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{y^3} \\
\frac{1}{y^3} \\
\frac{1}{y^3} \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{a^4} \\
\frac{1}{b^7} \\
\end{pmatrix}
\]

Questions:
1. What can you observe about the exponents of the given expressions?
2. How do you think these exponents are defined?
3. Do you think you can still apply your understanding of the laws of exponents to simplifying the given examples? Why?

The expressions above are expressions with rational exponents.

Review: Rational numbers are real numbers that can be written in the form \(\frac{a}{b}\), where \(a\) and \(b\) are integers and \(b \neq 0\). Hence, they can be whole numbers, fractions, mixed numbers, and decimals, together with their negative images.

➤ Activity 5: A New Kind of Exponent

You just reviewed the properties of integer exponents. Now, look at the expressions below. What could they mean? This activity will help us find out.

\[
\begin{pmatrix}
25^{\frac{1}{2}} \\
64^{\frac{1}{3}} \\
(-8)^{\frac{1}{3}} \\
(-1)^{\frac{1}{3}} \\
\end{pmatrix}
\]

Even though non-integral exponents have not been defined, we want the laws for integer exponents to also hold for expressions of the form \(b^{1/n}\). In particular, we want \((b^{1/n})^n = b\) to hold, even when the exponent of \(b\) is not an integer. How should \(b^{1/n}\) be defined so that this equation holds? To find out, fill up the following table. One row is filled up as an example.
<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^{1/n} )</td>
<td>( (b^{1/n})^n )</td>
<td>Value(s) of ( b^{1/n} ) that satisfy the equation in Column B</td>
</tr>
<tr>
<td>25(^{1/2})</td>
<td>((25^{1/2})^2 = 25)</td>
<td>5 and -5</td>
</tr>
<tr>
<td>64(^{1/3})</td>
<td>((-8)^{1/3})</td>
<td></td>
</tr>
<tr>
<td>((-8)^{1/3})</td>
<td>((-1)^{1/2})</td>
<td></td>
</tr>
</tbody>
</table>

The values in Column C represent the possible definitions of \( b^{1/n} \) such that rules for integer exponents may still hold. Now we will develop the formal definition for \( b^{1/n} \).

**Questions:**

1. When is there a unique possible value of \( b^{1/n} \) in Column C?
2. When are there no possible values of \( b^{1/n} \) in Column C?
3. When are there two possible values of \( b^{1/n} \) in Column C?
4. If there are two possible values of \( b^{1/n} \) in Column C, what can you observe about these two values?

If \( b^{1/n} \) will be defined, it has to be a unique value. If there are two possible values, we will define \( b^{1/n} \) to be the positive value.

Let us now consolidate our results below.

Recall from Grade 7 that if \( n \) is a positive integer, then \( \sqrt[n]{b} \) is the principal \( n \)th root of \( b \). We define \( b^{1/n} = \sqrt[n]{b} \), for positive integers \( n \).

For example,

1. \( 25^{1/2} = \sqrt{25} = 5, \text{ not } -5 \).
2. \( (-8)^{1/3} = \sqrt[3]{-8} = -2. \)
3. \( (-81)^{1/4} \) is not defined.

➤ Activity 6: Extend Your Understanding!

In this activity, you will learn the definition of \( b^{m/n} \). If we assume that the rules for integer exponents can be applied to rational exponents, how will the following expressions be simplified? One example is worked out for you.

1. \( (6^{1/2}) (6^{1/2}) = 6^{1/2} + 6^{1/2} = 6^1 = 6 \)
2. \( (2^{1/3}) (2^{1/3}) (2^{1/3}) (2^{1/3}) (2^{1/3}) (2^{1/3}) (2^{1/3}) = \) _______________
3. \( (10^{1/2}) (10^{1/2}) (10^{1/2}) (10^{1/2}) = \) _______________
4. \((-4)^{1/7}(-4)^{1/7}(-4)^{1/7} = \) ________________

5. \(13^{-1/4}13^{-1/4}13^{-1/4}13^{-1/4}13^{-1/4} = \) ________________

Questions:
1. If rules for integer exponents are applied to rational exponents, how can you simplify \((b^{1/n})^m\)?

2. If rules for integer exponents are applied to rational exponents, how can you simplify \((b^{1/n})^{-m}\)?

Let us now consolidate our results below.

Let \(m\) and \(n\) be positive integers. Then \(b^{m/n}\) and \(b^{-m/n}\) are defined as follows.
1. \(b^{m/n} = (b^{1/n})^m\), provided that \(b^{1/n}\) is defined.
   
   Examples: \(81^{3/4} = (81^{1/4})^3 = 3^3 = 27\)
   
   \((-8)^{2/3} = [(-8)^{1/3}]^2 = (-2)^2 = 4\)
   
   \((-1)^{3/2}\) is not defined because \((-1)^{1/2}\) is not defined.

2. \(b^{-m/n} = \frac{1}{b^{m/n}}\), provided that \(b \neq 0\).

What to PROCESS

Your goal in this section is to learn and understand the key concepts of negative integral, zero and rational exponents.

➤ Activity 7: What’s Happening!

Complete the table below and observe the pattern.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
<th>Column D</th>
<th>Column E</th>
<th>Column F</th>
<th>Column G</th>
<th>Column H</th>
</tr>
</thead>
<tbody>
<tr>
<td>4^0</td>
<td>1</td>
<td>4^{-1}</td>
<td>1/4</td>
<td>4^{-2}</td>
<td>1/16</td>
<td>4^{-3}</td>
<td>1/64</td>
</tr>
<tr>
<td>3^0</td>
<td>3^{-1}</td>
<td>3^{-2}</td>
<td>3^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2^0</td>
<td>2^{-1}</td>
<td>2^{-2}</td>
<td>2^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1^0</td>
<td>1^{-1}</td>
<td>1^{-2}</td>
<td>1^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((\frac{1}{2})^0)</td>
<td>((\frac{1}{2})^{-1})</td>
<td>((\frac{1}{2})^{-2})</td>
<td>((\frac{1}{2})^{-3})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Questions:
1. What do you observe from column B?
2. What happens to the value of the expression if the exponent is equal to zero?
3. If a certain number is raised to zero, is the answer the same if another number is raised to zero? Justify your answer.
4. What do you observe from columns D, F, and H?
5. What can you say if an expression is raised to a negative integral exponent?
6. Do you think it is true for all numbers? Cite some examples.
7. Can you identify a pattern for expressions or numbers raised to zero exponent? What is your pattern?
8. What do you think is zero raised to zero (0^0)?
9. Can you identify a pattern for expressions or numbers raised to negative integral exponents? What is your pattern?

In the previous activity, you learned that if \( n \) is a positive integer, then \( a^{-n} = \frac{1}{a^n} \) and if \( a \) is any real number, then \( a^0 = 1 \).

Let us further strengthen that understanding by answering the next activity.

➤ Activity 8: I’ll Get My Reward!

You can get the treasures of the chest if you will be able to correctly rewrite all expressions without using zero or negative integral exponent.

Questions:
1. Did you get the treasures? How does it feel?
2. How did you simplify the given expressions?
3. What are the concepts/processes to remember in simplifying expressions without zero and negative integral exponents?
4. Did you encounter any difficulties while solving? If yes, what are your plans to overcome them?

5. What can you conclude in relation to simplifying negative integral and zero exponents?

In the previous activity, you were able to simplify expressions with zero and negative integral exponents. Let us try that skill in answering the next challenging activity.

➤ Activity 9: I Challenge You!

Hi there! I am the MATH WIZARD, I came here to challenge you. Simplify the following expressions. If you do these correctly, I will have you as my apprentice. Good luck!

\[
\begin{align*}
6e^0 + (11f)^0 - \frac{5}{g^0} & \quad (3^{-4} + 5^{-3})^{-1} \\
\frac{5(2a^{-1}b^3)^0}{10c^{-5}d^8e^{-8}} & \quad -5(m^{-4}n^{-3})^3 \\
\frac{-5(m^{-4}n^{-3})^3}{7(p^{-6}q^8)^{-4}} & \quad \left(\frac{3x^{-4}y^5z^{-2}}{9x^2y^{-8}z^{-3}}\right)^{-2}
\end{align*}
\]

Questions:
1. How did you apply your understanding of simplifying expressions with zero and negative integral exponents to solve the given problems?
2. What are the concepts/processes to be remembered in simplifying expressions with zero and negative integral exponents?

Were you challenged in answering the previous activity? Did you arrive at the correct answers? Well, then let us strengthen your skill in simplifying expressions with negative integral and zero exponents by answering the succeeding activity.
Activity 10: Am I Right!

Des and Richard were asked to simplify \( \frac{b^5}{b^{-3}} \). Their solutions and explanations are shown below.

<table>
<thead>
<tr>
<th>Des</th>
<th>Richard</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{b^5}{b^{-3}} = \frac{b^5}{\frac{1}{b^3}} = b^5 \cdot b^3 = b^8 )</td>
<td>( \frac{b^5}{b^{-3}} = b^{5-(-3)} = b^{5+3} = b^8 )</td>
</tr>
<tr>
<td>Des used the concept of the negative exponent then followed the rule of dividing fractions.</td>
<td>Richard used the law of exponent.</td>
</tr>
</tbody>
</table>

Question:
Who do you think is correct in simplifying the given expression? Justify your answer.

Activity 11: How Many…?

Analyze and solve the problem below.

A very young caterpillar may weigh only 12^{-2} grams. It is possible for it to grow 12^{4} times its body weight during its life cycle.

Questions:
1. How many grams can it reach during its life cycle?
2. How did you apply your understanding of exponents in solving the problem?
3. What necessary concepts/skills are needed to solve the problems?
4. What examples can you give that show the application of zero and negative integral exponents?
5. Can you assess the importance of exponents in solving real-life problems? How?

You were able to simplify expressions with negative integral and zero exponents. Let us now learn how to simplify expressions with rational exponents.

Activity 12: Two Sides of the Same Coin

Simplify the following expressions. If the expression is undefined, write “undefined.”

1. \( 49^{\frac{1}{2}} \)
2. \( 125^{\frac{1}{3}} \)
3. \( 1000^{\frac{1}{3}} \)
4. \( (-32)^{\frac{1}{5}} \)
5. \( (-64)^{\frac{1}{5}} \)
6. \( (-100)^{\frac{1}{2}} \)
7. \( (-4)^{\frac{1}{2}} \)
8. \( -81^{\frac{1}{4}} \)
The previous activity required you to apply that $b^{1/n}$ is defined as the principal nth root of $b$.

Let us further simplify expressions with rational exponents by answering the succeeding activities.

➤ **Activity 13: Follow Me!**

Fill in the missing parts of the solution in simplifying expressions with rational exponents. Then answer the process questions below.

1. \[ \left( m^{\frac{2}{3}} \right) \left( m^{\frac{4}{3}} \right) = m^{\frac{2}{3} + \frac{4}{3}} = m^{\frac{6}{3}} = m^{2} \]
2. \[ \left( k^{\frac{1}{2}} \right) \left( k^{\frac{3}{2}} \right) = k^{\frac{1}{2} + \frac{3}{2}} = k^2 \]
3. \[ \frac{a^{\frac{5}{3}}}{a^{\frac{2}{3}}} = a^{\frac{5}{3} - \frac{2}{3}} = a^{\frac{3}{3}} = a \]
4. \[ \left( \frac{y^{\frac{2}{3}}}{y^{\frac{1}{2}}} \right)^2 = \frac{y^{\frac{2}{3}}}{y^{\frac{1}{2}}} = y^{\frac{2}{3} - \frac{1}{2}} = y^{\frac{1}{6}} \]
5. \[ (r^{12}s^3)^{\frac{1}{3}} = r^{12 \cdot \frac{1}{3}}s^{3 \cdot \frac{1}{3}} = r^4s^1 \]

**Questions:**

1. Based on the activity, how do you simplify expressions involving rational exponents?
2. What are the necessary skills in simplifying expressions with rational exponents?
3. Did you encounter any difficulties while solving? If yes, what are your plans to overcome them?

The previous activities enabled you to realize that laws of exponents for integral exponents may be used in simplifying expressions with rational exponents.

Let $m$ and $n$ be rational numbers and $a$ and $b$ be real numbers.

\[
\begin{align*}
    a^m \cdot a^n &= a^{m+n} \\
    (a^m)^n &= a^{mn} \\
    (ab)^m &= a^m b^m \\
    \left( \frac{a}{b} \right)^m &= \frac{a^m}{b^m}, \quad b \neq 0 \\
    \frac{a^m}{b^m} &= a^{m-n}, \quad \text{if } m > n \\
    \frac{a^m}{b^m} &= \frac{1}{a^{m-n}}, \quad \text{if } m < n
\end{align*}
\]

Note: Some real numbers raised to a rational exponent, such as $(-1)^{\frac{1}{2}}$, are not real numbers. In such cases, these laws do not hold.

Aside from the laws of exponents, you were also required to use your understanding of addition and subtraction of similar and dissimilar fractions.

Answer the next activity that will strengthen your skill in simplifying expressions with rational exponents.
Activity 14: Fill-Me-In! (by dyad/triad)

Simplify the following expressions with rational exponents by filling in the boxes with solutions. Then answer the process questions that follow.

Questions:
1. How do you simplify expressions with rational exponents?
2. What are the needed knowledge and skills to remember in simplifying expressions with rational exponents?
3. Can you propose an alternative process in simplifying these expressions? How?
4. Have you encountered any difficulties while solving? If yes, what are your plans to overcome these difficulties?

Now that you are capable of simplifying expressions with rational exponents by using the laws of integral exponents, let us put that learning to the test through answering the succeeding activities.
Activity 15: Make Me Simple!

Using your knowledge of rational expressions, simplify the following.

<table>
<thead>
<tr>
<th>Given</th>
<th>Final Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left( \frac{k^\frac{2}{3}}{k^\frac{6}{7}} \right) )</td>
<td>1.</td>
</tr>
<tr>
<td>( \left( x^{16} y^{20} z^8 \right)^\frac{1}{3} )</td>
<td>2.</td>
</tr>
<tr>
<td>( \left( p^{21} q^{-15} r^{-3} \right)^\frac{1}{3} )</td>
<td>3.</td>
</tr>
<tr>
<td>( \frac{m^{\frac{1}{3}} n^{\frac{1}{7}}}{m^{\frac{1}{5}} n^{\frac{1}{7}}} )</td>
<td>4.</td>
</tr>
<tr>
<td>( \frac{3 x^\frac{3}{2} y^{\frac{1}{4}}}{x^{\frac{5}{2}} y^{\frac{3}{2}}} )</td>
<td>5.</td>
</tr>
</tbody>
</table>

Questions:
1. How did you simplify the given expressions?
2. How would you simplify expressions with positive integral exponents? expressions with negative integral exponents?
3. What mathematical concepts are important in simplifying expressions with rational exponents?

You just tested your understanding of the topic by answering the series of activities given to you in the previous section. Let us now try to deepen that understanding in the next section.

What to REFLECT and UNDERSTAND

Your goal in this section is to take a closer look at some aspects of the topic. Hope that you are now ready to answer the exercises given in this section. The activities aim to intensify the application of the different concepts you have learned.
Activity 16: Tke-It-2-D-Nxt-Lvl!

Solve the given problem then answer the process questions.

\[ 10^{-3} + 10^{-2} + 10^1 + 10^0 + 10^{-1} + 10^{-2} \]

\[ 10^{-3} + 25^2 - 8^3 \cdot 4^2 + 27^0 \]

Questions:
1. What is your final answer in the first problem? in the second problem?
2. What approach did you use to arrive at your answers?
3. Are there concepts/processes to strictly follow in solving the problem?
4. How would you improve your skill in simplifying these expressions?
5. How can you apply the skills/concepts that you learned on exponents to real-life situations?

In the previous activity, you were able to simplify expressions with rational, negative integral, and zero exponents all in one problem. Moreover, you were able to justify your idea by answering the questions that follow. Did the previous activity challenge your understanding on simplifying zero, negative integral, and rational exponents? How well did you perform? Let us deepen that understanding by answering some problems related to the topic.

Activity 17: How Many…?

Solve the following problem.

A seed on a dandelion flower weighs $15^{-3}$ grams. A dandelion itself can weigh up to $15^3$ grams. How many times heavier is a dandelion than its seeds?

Questions:
1. How did you apply your understanding of exponents in solving the problem?
2. What necessary concepts/skills are needed to solve the problems?
3. What examples can you give that show the application of zero and negative integral exponents?
4. Can you assess the importance of exponents in solving real life problems? How?
Activity 18: Create a Problem for Me?

Formulate a problem based on the illustration and its corresponding attribute. Show your solution and final answer for the created problem. Your work shall be evaluated according to the rubric.

Given: time taken by light to travel 1 meter is roughly $3 \times 10^{-8}$ seconds

Formulated problem and solution:

Given: diameter of the atomic nucleus of a lead atom is $1.75 \times 10^{-15}$ m

Formulated problem and solution:

Given: the charge on an electron is roughly $1.6 \times 10^{-19}$ coulombs

Formulated problem and solution:

Given: period of a 100 MHz FM radio wave is roughly $1 \times 10^{-8}$ s

Formulated problem and solution:
### Rubrics for the Task (Create A Problem For Me?)

<table>
<thead>
<tr>
<th>Categories</th>
<th>4 Excellent</th>
<th>3 Satisfactory</th>
<th>2 Developing</th>
<th>1 Beginning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical Concept</strong></td>
<td>Demonstrate a thorough understanding of the topic and use it appropriately to solve the problem.</td>
<td>Demonstrate a satisfactory understanding of the concepts and use it to simplify the problem.</td>
<td>Demonstrate incomplete understanding and have some misconceptions.</td>
<td>Show lack of understanding and have severe misconceptions.</td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td>The computations are accurate and show a wise use of the key concepts of zero, negative, and rational exponents.</td>
<td>The computations are accurate and show the use of key concepts of zero, negative, and rational exponents.</td>
<td>The computations are erroneous and show some use of the key concepts of zero, negative, and rational exponents.</td>
<td>The computations are erroneous and do not show the use of key concepts of zero, negative, and rational exponents.</td>
</tr>
</tbody>
</table>

**Questions:**
1. How did you formulate the problems? What concepts did you take into consideration?
2. How can you apply the skills/concepts that you learned on this activity in real-life situation?

Was it easy for you the formulate real-life problems involving negative integral exponents? How did you apply your understanding in accomplishing this activity?
Since you are now capable of simplifying these exponents, let us revisit and answer the Anticipation-Reaction Guide that you had at the beginning of this module.
Activity 19: Agree or Disagree! (revisited)

Read each statement under the column Statement, then write A if you agree with the statement; otherwise, write D. Write your answer on the “Response-After-the-Discussion” column.

Anticipation-Reaction Guide

<table>
<thead>
<tr>
<th>Response-Before-the-Discussion</th>
<th>Statement</th>
<th>Response-After-the-Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any number raised to zero is equal to one (1).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>An expression with a negative exponent CANNOT be written into an expression with a positive exponent.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^{-3}$ is equal to $\frac{1}{8}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laws of exponents may be used in simplifying expressions with rational exponents.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\left(\frac{1}{3}\right)^{-2} = 9$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3^04^{-2} = 16$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\left(32x^3y^5\right)^{-2}}$ may be written as $(32x^3y^5)^2$ where $x \neq 0$ and $y \neq 0$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-16)^{\frac{3}{2}} = -16$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The exponential expression $\frac{1}{(x + 10)^{\frac{1}{2}}}$ equivalent to $x^{\frac{1}{2}} + 10^{\frac{1}{2}}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3^{\frac{1}{2}} \cdot 4^0 + 1^{\frac{1}{2}} \cdot 5^0 = 11$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Questions:

1. Is there any change in your answer from the “Response-Before-the-Discussion” column to the “Response-After-the-Discussion” column? Why?
2. Based on your understanding, how would you explain the use of the laws of exponents in simplifying expressions with rational exponents?
3. What examples can you give that show the importance of expressions with negative and rational exponents?
Were you able to answer the preceding activities correctly? Which activity interests you the most? What activity did you find difficult to answer? How did you overcome these difficulties?
Let us have some self-assessment first before we proceed to the next section.

➤ Activity 20: 3-2-1 Chart

Fill-in the chart below.

<table>
<thead>
<tr>
<th>3 things I learned</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2 things that interest me</td>
<td></td>
</tr>
<tr>
<td>1 application of what I learned</td>
<td></td>
</tr>
</tbody>
</table>

Now that you better understand zero, negative integral and rational exponents, let us put that understanding to the test by answering the transfer task in the next section.

What to TRANSFER

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding. This task challenges you to apply what you learned about zero, negative integral and rational exponents. Your work will be graded in accordance to the rubric presented.
Activity 21: Write about Me!

A math magazine is looking for new and original articles for its edition on the topic Zero, Negative, and Rational Exponents Around Us. As a freelance researcher/writer, you will join the said competition by submitting your own article/feature. The output will be evaluated by the chief editor, feature editor, and other writers of the said magazine. They will base their judgment on the accuracy, creativity, mathematical reasoning, and organization of the report.

Rubrics for the Performance Task

<table>
<thead>
<tr>
<th>Categories</th>
<th>4 Excellent</th>
<th>3 Satisfactory</th>
<th>2 Developing</th>
<th>1 Beginning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Concept</td>
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<td>Demonstrate incomplete understanding and have some misconceptions.</td>
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<tr>
<td>Accuracy</td>
<td>The computations are accurate and show a wise use of the key concepts of zero, negative and rational exponents.</td>
<td>The computations are accurate and show the use of key concepts of zero, negative and rational exponents.</td>
<td>The computations are erroneous and show some use of the key concepts of zero, negative and rational exponents.</td>
<td>The computations are erroneous and do not show the use of key concepts of zero, negative and rational exponents.</td>
</tr>
</tbody>
</table>
Activity 22: **Synthesis Journal**

Complete the table below by answering the questions.

<table>
<thead>
<tr>
<th>How do I find the performance task?</th>
<th>What are the values I learned from the performance task?</th>
<th>How did I learn them? What made the task successful?</th>
<th>How will I use these learning/insights in my daily life?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is the end of Lesson 1: Zero, Negative Integral, and Rational Exponents of Module 4: Radicals. Do not forget your what have you learned from this lesson for you will use this to successfully complete the next lesson on radicals.

**Summary/Synthesis/Generalization:**

This lesson was about zero, negative integral, and rational exponents. The lesson provided you with opportunities to simplify expressions with zero, negative integral, and rational exponents. You learned that any number, except 0, when raised to 0 will always result in 1, while expressions with negative integral exponents can be written with a positive integral exponent by getting the reciprocal of the base. You were also given the chance to apply your understanding of the laws of exponents to simplify expressions with rational exponents. You identified and described the process of simplifying these expressions. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the next lesson on radicals.
What to **KNOW**

What is the connection between expressions with rational exponents and radicals? Why do we need to know how to simplify radicals? Are radicals really needed in life outside math studies? **How can you simplify radical expressions? How do you operate with radicals? How can the knowledge of radicals help us solve problems in daily life?**

In this lesson we will address these questions and look at some important real-life applications of radicals.

➤ **Activity 1: Let’s Recall**

Simplify the following expressions.

1. \( \left( \frac{2}{5} \right)^{\frac{2}{3}} \cdot \left( \frac{5}{3} \right) \)
2. \( \left( x^{16} y^{0} z^{8} \right)^{\frac{1}{8}} \)
3. \( \left( \frac{5}{1} \right)^{24} \)
4. \( \frac{1}{\sqrt[1]{m^{2} n^{-7}}} \)
5. \( \left( -3e^{4} \right) \left( \frac{f^5}{f^0} \right) \)

Questions:

1. How did you solve the problem?
2. What important concepts/skills are needed to solve the problem?

Did you answer the given problem correctly? Can you still recall the laws of exponents for zero, negative integral, and rational exponents? Did you use them to solve the given problem? The next activity will elicit your prior knowledge regarding this lesson.

➤ **Activity 2: IRF Sheet**

Below is an Initial-Revise-Final Sheet. It will help check your understanding of the topics in this lesson. You will be asked to fill in the information in different sections of this lesson. For now you are supposed to complete the first column with what you know about the topic.

<table>
<thead>
<tr>
<th>Initial</th>
<th>Revise</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are your initial ideas about radicals?</td>
<td>do not answer this part yet</td>
<td>do not answer this part yet</td>
</tr>
</tbody>
</table>
The previous activities helped you recall how to simplify expressions with zero, negative integral, and rational exponents. These also elicited your initial ideas about radicals. Were you able to answer the problem correctly?

Answer the next activity that will require you to write expressions with rational exponents as radicals and vice versa.

What to PROCESS

Your goal in this section is to construct your understanding of writing expressions with rational exponents to radicals and vice versa, simplifying and operating radicals.

Towards the end of this module, you will be encouraged to apply your understanding of radicals to solving real-life problems.

➤ Activity 3: Fill–Me–In

Carefully analyze the first two examples below then fill in the rest of the exercises with the correct answer.

\[
\begin{align*}
\frac{2}{3^2} &\rightarrow \sqrt{3^2} &\rightarrow \sqrt{9} \\
(2n)^{\frac{3}{2}} &\rightarrow \sqrt[3]{2^3 n^3} &\rightarrow \sqrt[3]{8n^3} \\
\frac{3}{5^3} &\rightarrow \cdot \\
\frac{3}{3b^2} &\rightarrow \cdot \\
\left(\frac{3}{2p^2}\right)^{\frac{2}{3}} &\rightarrow \cdot \\
\frac{(x^2 + 3)^{\frac{1}{3}}}{(x^2 - 3)^{\frac{1}{3}}} &\rightarrow \cdot 
\end{align*}
\]
<table>
<thead>
<tr>
<th>Questions</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do you think the given expressions with a rational exponent were written as radicals? What processes have you observed?</td>
<td></td>
</tr>
<tr>
<td>What necessary understanding is needed to simplify the given expression?</td>
<td></td>
</tr>
<tr>
<td>What are the bases for arriving at your conclusion?</td>
<td></td>
</tr>
</tbody>
</table>
Let us consolidate the results below.

The symbol $\sqrt[n]{a^m}$ is called radical. A **radical expression** or a **radical** is an expression containing the symbol $\sqrt{}$ called **radical sign**. In the symbol $\sqrt[n]{a^m}$, $n$ is called the **index** or **order** which indicates the degree of the radical such as square root $\sqrt{}$, cube root $\sqrt[3]{\text{ }}$ and fourth root $\sqrt[4]{\text{ }}$, $a^m$ is called the **radicand** which is a number or expression inside the radical symbol and $m$ is the power or exponent of the radicand.

If $\frac{m}{n}$ is a rational number and $a$ is a positive real number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

provided that $\sqrt[n]{a^m}$ is a real number. The form $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$ is called the principal $n$th root of $a^m$. Through this, we can write expressions with rational exponents as radicals.

Examples:

$2^{\frac{1}{2}} = \sqrt{2} = \sqrt{2}$

$(3a)^{\frac{2}{3}} = \sqrt[3]{(3a)^2} = \sqrt[3]{3^2a^2} = \sqrt[3]{9a^2}$

Note: We need to impose the condition that $a > 0$ in the definition of $\sqrt[n]{a^m}$ for an even $n$ because it will NOT hold true if $a < 0$. If $a$ is a negative real number and $n$ is an even positive integer, then $a$ has NO real $n$th root.

If $a$ is a positive or negative real number and $n$ is an odd positive integer, then there exists exactly one real $n$th root of $a$, the sign of the root being the same as the sign of the number.

Examples:

$\sqrt{-8} = \text{no real root}$

$\sqrt{-8} = -2$

$\sqrt[3]{-32} = \text{no real root}$

$\sqrt[3]{-32} = -2$

Answer the next activity that will test your skill in writing expressions with rational exponents to radicals and vice versa.

➤ **Activity 4: Transformers I**

Transform the given radical form into exponential form and exponential form into radical form. Assume that all the letters represent positive real numbers.

<table>
<thead>
<tr>
<th>Radical Form</th>
<th>Exponential Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{11x^2}$</td>
<td>$(11x^2)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>$(26)^{\frac{1}{2}}$</td>
<td>$\sqrt{26}$</td>
</tr>
<tr>
<td>$(5ab^2)^{\frac{2}{3}}$</td>
<td>$\sqrt[3]{(5ab)^2}$</td>
</tr>
<tr>
<td>$\sqrt[3]{\frac{3}{y^2}}$</td>
<td>$(\frac{3}{y^2})^{\frac{1}{3}}$</td>
</tr>
</tbody>
</table>
Questions:
1. How did you answer the given activity?
2. What are the necessary concepts/processes needed in writing expressions with rational exponents as radicals?
3. At which part of the process are the laws of exponents necessary?
4. What step-by-step process can you create on how to write expressions with rational exponents as radicals? radicals as expressions with rational exponents?
5. Have you encountered any difficulties while rewriting? If yes, what are your plans to overcome them?

In the previous lesson, you learned that $a^{1/n}$ is defined as the principal $n$th root of $b$. In radical symbols: $\sqrt[n]{a} = a^{\frac{1}{n}}$; and for $a > 0$ and positive integers $m$ and $n$ where $n > 1$, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$, provided that it is defined.

Using this knowledge, did you correctly answer most of the problems in the previous activity? You will need those skills to succeed in the next activity.

---

**Activity 5: The Pair Cards (Group Activity)**

**Mechanics of the Game**
1. You will be playing “The Pair Cards” game similar to a well-known card game, “Unggoyan.”
2. Every group shall be given cards. Select a **dealer**, who is at the same time a player, to facilitate the distribution of cards. There must be at most 10 cards in every group. (Note: There should be an even number of cards in every group.)
3. After receiving the cards, pair the expressions. A pair consists of a radical expression and its equivalent expression with a rational exponent. Then, place and reveal the paired cards in front.
4. If there will be no paired cards left with each player, the dealer will have the privilege to be the first to pick a card from the player next to him following a clockwise direction. He/she will then do step 3. This process will be done by the next players one at a time.
5. The game continues until all the cards are paired.
6. The group who will finish the game ahead of others will be declared the “WINNER!”

---

<table>
<thead>
<tr>
<th>$-29 \sqrt[4n]{\frac{m^4 p}{4}}$</th>
<th>$(4r^2 s^4 t^4)^\frac{2}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x^6}{4\sqrt[4]{y^4}}$</td>
<td>$3 \div 4\sqrt[4]{b^4}$</td>
</tr>
<tr>
<td></td>
<td>$-(3k^2)^\frac{2}{7}$</td>
</tr>
<tr>
<td></td>
<td>$(\frac{4a^4}{5b^3})^{\frac{3}{7}}$</td>
</tr>
</tbody>
</table>
Examples of expressions in the card;

Source (Modified): Beam Learning Guide, Second Year – Mathematics, Module 10: Radical Expressions in General, pages 31-33

Questions:
1. Did your group win this activity? How did you do it?
2. What skills are needed to correctly answer the problems in this activity?
3. How would you compare the ideas that you have with your classmates’ ideas?
4. What insights have you gained from this activity?

Winning in the previous activity means you are now really capable of writing expressions with rational exponents into radicals and vice versa. Losing would mean there is a lot of room for improvement. Try to ask your teacher or peer about how to improve this skill.

Since you are now capable of writing expressions with rational exponents as radicals, let us now learn how to simplify radical expressions through the following laws on radicals. Assume that when $n$ is even, $a > 0$.

a. $(\sqrt[n]{a})^n = a$
   
   Examples: $(\sqrt[3]{4})^3 = 4$  \(\sqrt[3]{64} = \sqrt[3]{8^2} = 8\)

b. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
   
   Examples: $\sqrt[3]{50} = \sqrt[3]{25 \cdot 2} = 5\sqrt[3]{2}$  \(\sqrt[3]{-32x^3} = \sqrt[3]{-2^3x^3} \cdot \sqrt[3]{2^2x^2} = -2x\sqrt[3]{4x^2}\)

c. $\sqrt[n]{a} = \sqrt[n]{a}$, $b > 0$
   
   Examples: $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$  \(\sqrt[3]{9} = \sqrt[3]{3^2} = \frac{x^{12}}{3}\)

d. $\sqrt[n]{a} = \frac{\sqrt[n]{a}}{\sqrt[n]{a}}$
   
   Examples: $\sqrt[3]{4} = \sqrt[3]{2^2} = \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$  \(\sqrt[3]{27} = \sqrt[3]{3^3} = \sqrt[3]{3}\)

Simplifying Radicals:

a. Removing Perfect $n$th Powers

Break down the radicand into perfect and nonperfect $n$th powers and apply the property $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.  

Example: $\sqrt[3]{30x^3} = \sqrt[3]{(30x^2) \cdot x} = \sqrt[3]{30x^2} \cdot \sqrt[3]{x}$
**Example:** \( \sqrt{8x^3y^6z^{13}} = \sqrt{2^3(x^3)(y^6)(z^{13})} \cdot \sqrt{2xz} = 2x^2y^3z^6 \sqrt{2xz} \)

**b. Reducing the index to the lowest possible order**

Express the radical into an expression with a rational exponent then simplify the exponent or apply the property \( a^{\frac{m}{n}} = \sqrt[n]{a^m} \).

Examples:

\[
\sqrt[3]{32m^{15}n^5} = \sqrt[3]{(2^5)(m^5)^3} = \sqrt[3]{2m^n} \quad \text{or} \quad \sqrt[3]{32m^{15}n^5} = (2^5m^{15}n^5)^{\frac{1}{20}} = \frac{5}{20}m^{\frac{15}{20}}n^{\frac{5}{20}} = \frac{1}{2}m^\frac{3}{4}n^{\frac{1}{4}} = (2m^n)^{\frac{1}{2}} = \sqrt{2m^n}
\]

**c. Rationalizing the denominator of the radicand**

**Rationalization** is the process of removing the radical sign in the denominator.

Examples:

\[
\sqrt{\frac{3}{4k}} = \sqrt{\frac{3 \cdot 2^2}{2^2k}} = \sqrt{\frac{6k^2}{2^3k^3}} = \sqrt{\frac{6k^2}{2k}}
\]

\[
\sqrt{\frac{1}{72}} = \sqrt{\frac{1}{36 \cdot 2}} = \sqrt{\frac{1 \cdot 6^2 \cdot 2^2}{6^3 \cdot 2^3}} = \sqrt{\frac{288}{6^4 \cdot 2^4}} = \sqrt{\frac{18 \cdot 16}{6^4 \cdot 2^4}} = \sqrt{\frac{18 \cdot 2^4}{6^4 \cdot 2^4}} = \frac{2\sqrt{18}}{6^2} = \frac{2\sqrt{18}}{36} = \frac{\sqrt{18}}{6}
\]

**The simplified form of a radical expression** would require:

- NO prime factor of a radicand that has an exponent equal to or greater than the index.
- NO radicand contains a fraction.
- NO denominator contains a radical sign.

Let us try your skill in simplifying radicals by answering the succeeding activities.

---

**Activity 6: Why Am I True/Why Am I False?**

Given below are examples of how to simplify radicals. Identify if the given process below is TRUE or FALSE, then state your reason. For those you identified as false, make it true by writing the correct part of the solution.

<table>
<thead>
<tr>
<th>True or False</th>
<th>Why?</th>
<th>If false, write the correct part of the solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplify ( \sqrt{16} )</td>
<td>( \sqrt{16} = \sqrt[6]{8} \cdot \sqrt{2} )</td>
<td>( \sqrt{16} = \sqrt[6]{2^3} \cdot \sqrt{2} )</td>
</tr>
<tr>
<td></td>
<td>( = \sqrt[6]{2^3} \cdot \sqrt{2} )</td>
<td>( = 2 \cdot \sqrt{2} )</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{16} = 2\sqrt{2} )</td>
<td></td>
</tr>
</tbody>
</table>
### Simplify, $\sqrt[3]{m^{12}}$ where $m > 0.$

$\sqrt[3]{m^{12}} = (m^{12})^{\frac{1}{3}} = m^{4}$

$= m^{3}$

$\sqrt[3]{m^{12}} = \sqrt[3]{m^{3}}$

### Simplify, $\sqrt[6]{\frac{1}{2^s}}$ where $s \neq 0.$

$\sqrt[6]{\frac{1}{2^s}} = \sqrt[6]{\frac{1}{2^s} \cdot 2^s s^s} = \sqrt[6]{2 s^s} = \sqrt[6]{2^s s^6} = \sqrt[6]{32 s^5} \cdot \frac{1}{2s}$

### Questions:

1. How do you think the given expressions were simplified? What processes have you observed?
2. How do we simplify radicals with the same index?
3. How do we simplify radicals with different indices?
4. How do we simplify expressions with radicals in the denominator?
5. What important understanding is necessary to simplify the given expression?

In the previous activity, you were able to simplify radicals by reducing the radicand, by reducing the order of the radical, and simplifying radicals by making the order the same. Were you able to identify which part of the process is true or false? Have you determined the reason for each process? Let us put that knowledge to the test by decoding the next activity.
Activity 7: Who Am I?

Using your knowledge of rational exponents, decode the following.

The First Man to Orbit the Earth

In 1961, this Russian cosmonaut orbited the earth in a spaceship. Who was he? To find out, evaluate the following. Then encircle the letter that corresponds to the correct answer. These letters will spell out the name of this Russian cosmonaut. Have fun!

1. \(144^{\frac{1}{2}}\) Y. 12 Z. 14
2. \(169^{\frac{1}{2}}\) O. 9 U. 13
3. \(-\left(49^{\frac{1}{2}}\right)\) Q. 25 B. -5
4. \(216^{\frac{1}{3}}\) E. 16 I. 6
5. \(625^{\frac{1}{4}}\) G. 5 H. 25
6. \(9^{\frac{2}{3}}\) A. 27 M. -9
7. \(25^{\frac{1}{3}}\) F. -4 X. 4
8. \(9(27)^{\frac{1}{3}}\) S. 81 J. -81
9. \((-343)^{\frac{1}{3}}\) R. -7 S. 7
10. \(36^{\frac{1}{2}}\) L. -16 D. 16
11. \(-\left(\frac{16}{81}\right)^{\frac{3}{4}}\) N. \(-\frac{8}{27}\) P. \(\frac{8}{27}\)

Answer:

1 2 3 4 5 6 7 8 9 10 11

Source (Modified): EASE Modules, Year 2 – Module 2 Radical Expressions, pages 9–10

Questions:
1. How did you solve the given activity?
2. What mathematical concepts are important in simplifying expressions with rational exponents?
3. Did you encounter any difficulties while solving? If yes, what are your plans to overcome those difficulties?

Now that you are knowledgeable in simplifying radicals, try to develop your own conclusion about it.
Activity 8: Generalization

Write your generalization on the space provided regarding simplifying radicals.

We can simplify radicals...

You are now capable of simplifying radicals by removing the perfect \( n \)th power, reducing the index to the lowest possible order and rationalizing the denominator of the radicand. Let us put those skills into a higher level through an operation on radical expressions.

Carefully analyze the given examples below. In the second example, assume that \( y > 0 \). Then complete the conclusion table.

- Add or subtract as indicated.

\[
5\sqrt{6} + 9\sqrt{6} - 8\sqrt{6} + 11\sqrt{6} = (5 + 9 - 8 + 11)\sqrt{6} = 17\sqrt{6}
\]

- Add or subtract by combining similar radicals.

\[
20\sqrt{x} - 10\sqrt{y} + \sqrt{y} - 5\sqrt{x} = (20 - 5)\sqrt{x} + (-10 + 1)\sqrt{y} = 15\sqrt{x} - 9\sqrt{y}
\]

- Some radicals have to be simplified before they are added or subtracted.

\[
\frac{3}{2} + \sqrt{24} = \frac{3}{2} \cdot \frac{1}{g} + \sqrt{4 \cdot 6} = \frac{1}{2} + \sqrt{6} = \frac{2}{2} + \sqrt{6} = \left(\frac{1}{2} + 2\right)\sqrt{6} = \frac{5}{2}\sqrt{6}
\]

<table>
<thead>
<tr>
<th>CONCLUSION TABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions</td>
</tr>
<tr>
<td>How do you think the given expressions were simplified? What processes have you observed?</td>
</tr>
<tr>
<td>What understanding is necessary to simplify the given expression?</td>
</tr>
<tr>
<td>Based on the given illustrative examples, how do we add radicals? How do we subtract radicals? What conclusion can you formulate regarding addition and subtraction of radicals?</td>
</tr>
<tr>
<td>What are your bases for arriving at your conclusion?</td>
</tr>
</tbody>
</table>
Let us consolidate your answers:

In the previous activity, you were able to develop the skills in adding and subtracting radicals. Take note of the kinds of radicals that can be added or subtracted. **Similar radicals** are radicals of the same order and the same radicand. These radicals can be combined into a single radical. Radicals of different indices and different radicands are called **dissimilar radicals**. Answer the next activity that deals with this understanding.

**Activity 9: Puzzle-Math**

Perform the indicated operation/s as you complete the puzzle below.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3\sqrt{2} )</td>
<td>+</td>
<td>( 5\sqrt{2} )</td>
<td>=</td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>( 10\sqrt{6} )</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>=</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td>(-10\sqrt{2} = 5\sqrt{6})</td>
<td>+</td>
<td>=</td>
<td></td>
</tr>
<tr>
<td></td>
<td>=</td>
<td></td>
<td>=</td>
</tr>
<tr>
<td>(-24\sqrt{2})</td>
<td>-</td>
<td>(-20\sqrt{6})</td>
<td>-</td>
</tr>
</tbody>
</table>

**Questions:**
1. How is addition or subtraction of radicals related to other concepts of radicals?
4. How can you apply this skill to real-life situations?
5. Did you encounter any difficulties while solving? If yes, what are your plans to overcome those difficulties?

The previous activity deals with addition and subtraction of radicals. You should know by now that only similar radicals can be added or subtracted. Recall that **similar radicals** are radicals with the same index and radicand. We only **add or subtract the coefficients then affix the common radical**.

Let us now proceed to the next skill which is multiplication of radicals.

**Activity 10: Fill-in-the-Blanks**

Provided below is the process of multiplying radicals where \( x > 0 \) and \( y > 0 \). Carefully analyze the given example then provide the solution for the rest of the problems. Then answer the conclusion table that follows.
Let us consolidate your conclusion below.

How was your performance in multiplying radicals? Were you able to arrive at your own conclusions?

**a) To multiply radicals of the same order**, use the property $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, then simplify by removing the perfect nth powers from the radicand.

Example: $\sqrt[3]{r^3s^5t^7} \cdot \sqrt[3]{12r^5s^2t^6} = \sqrt[3]{36r^8s^7t^{13}} = 6^2 \cdot (r^3) \cdot (s^7) \cdot (t^{13}) \cdot \sqrt[3]{12}$

**b) To multiply binomials involving radicals**, use the property for the product of two binomials $(a \pm b)(c \pm d) = ac(ad \pm bc) \pm bd$, then simplify by removing perfect nth powers from the radicand or by combining similar radicals.
Example: \((\sqrt{2} + \sqrt{7})(\sqrt{8} - \sqrt{6}) = \sqrt{2} \cdot 8 - \sqrt{2} \cdot 6 + \sqrt{7} \cdot 8 - \sqrt{7} \cdot 6\)
\[= \sqrt{16} - \sqrt{12} + \sqrt{56} - \sqrt{42}\]
\[= \sqrt{4^2} - \sqrt{2^2 \cdot 3} + \sqrt{2^2 \cdot 14} - \sqrt{42}\]
\[= 4 - 2\sqrt{3} + 2\sqrt{14} - \sqrt{42}\]

C) To multiply radicals of different orders, express them as radicals of the same order then simplify.

Example: \(\sqrt[4]{4} \cdot \sqrt[2]{3} = \sqrt[4]{2^2} \cdot \sqrt[2]{2} = 2^{\frac{2}{4}} \cdot 2^\frac{1}{2} = 2^{\frac{2}{4} + \frac{1}{2}} = 2^{\frac{3}{4}} = \sqrt[4]{8}\)

Let us now proceed to the next activities that apply your knowledge of multiplying radicals.

**Activity 11: What's the Message?**

*Do you feel down even with people around you? Don't feel low.* Decode the message by performing the following radical operations. Write the words corresponding to the obtained value in the box provided.

<table>
<thead>
<tr>
<th>are not</th>
<th>(\sqrt{2} \cdot 5\sqrt{8})</th>
<th>for people</th>
<th>((4\sqrt{3a^3})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>and irreplaceable</td>
<td>(\sqrt{7} \cdot \sqrt{7})</td>
<td>is unique</td>
<td>(\sqrt{3} \cdot \sqrt{18})</td>
</tr>
<tr>
<td>consider yourself</td>
<td>(4\sqrt{3} \cdot 3\sqrt{3})</td>
<td>more or less</td>
<td>(\sqrt{27} \cdot \sqrt{3})</td>
</tr>
<tr>
<td>Do not</td>
<td>(\sqrt{9} \cdot \sqrt{4})</td>
<td>nor even equal</td>
<td>(\sqrt{a} \left(\sqrt{a^3} - 7\right))</td>
</tr>
<tr>
<td>Each one</td>
<td>(\sqrt[3]{9x^2} \cdot 3\sqrt[3]{3x^4y^6})</td>
<td>of identical quality</td>
<td>(5\sqrt{7} \cdot 2\sqrt{7})</td>
</tr>
<tr>
<td>to others</td>
<td>(\sqrt[5]{a} \left(\sqrt[2]{a}\right) \left(3\sqrt[10]{a^2}\right))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 6 | 36 | 9 | \(a^2 - 7\sqrt{a}\) | 30a^2 | 48a^3 |
| 20 | 70 | \(9xy^2 \sqrt{x^2y^2}\) | \(3\sqrt[12]{12}\) | \(\sqrt[8]{23543}\) |

*Source (Modified): EASE Modules, Year 2-Module 5 Radical Expressions page 10*
You now know that in multiplying radicals of the same order, we just multiply its radicands then simplify. If the radicals are of different orders, we transform first to radicals with same indices before multiplying. Your understanding of the property for the product of two binomials can be very useful in multiplying radicals.

How well have you answered the previous activity? Were you able to answer majority of the problems correctly? Well then, let’s proceed to the next skill.

As you already know in simplifying radicals there should be NO radicals in the denominator. In this section, we will recall the techniques on how to deal with radicals in the denominator.

Carefully analyze the examples below. Perform the needed operations to transform the expression on the left to its equivalent on the right. The first problem is worked out for you.

Questions:
1. How can we simplify radicals if the denominator is of the form \(\sqrt{a}\)?
2. How do you identify the radical to be multiplied to the whole expression?

In the previous activity, you were able to simplify the radicals by rationalizing the denominator. Review: Rationalization is a process where you simplify the expression by making the denominator free from radicals. This skill is necessary in the division of radicals.

a) To divide radicals of the same order, use the property \(\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\) then rationalize the denominator.

Examples:
- \(\frac{\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}} = \sqrt{2.5} = \sqrt{2.5} = \sqrt{20}\)
- \(\frac{\sqrt{1}}{\sqrt{ab}} = \sqrt{\frac{1}{ab}} = \sqrt{\frac{a^2}{ab}} = \sqrt{\frac{a^2}{ab}} = \sqrt{\frac{a^2b}{ab}} = \sqrt{\frac{ab}{ab}}

b) To divide radicals of different orders, it is necessary to express them as radicals of the same order then rationalize the denominator.

Examples:
- \(\frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \sqrt[3]{\frac{3}{3}} = \sqrt[3]{1} = \sqrt[3]{1} = \sqrt[3]{1}\)
- \(\frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \sqrt[3]{\frac{3}{3}} = \sqrt[3]{1} = \sqrt[3]{1} = \sqrt[3]{1}\)

Now let’s consider expressions with two terms in the denominator.
Carefully analyze the examples below. What is the missing factor? The first problem is worked out for you.

| \((\sqrt{2} + 3)\) | \((\sqrt{2} + 3)(\sqrt{2} - 3)\) | 2 – 9 | –7 |
| \((\sqrt{6} - 5)\) | \((\sqrt{6} - 5)(\_\_\_)\) | 6 – 25 | \_\_\_\_\_ |
| \((\sqrt{2} + \sqrt{3})\) | \((\sqrt{2} + \sqrt{3})(\_\_\_)\) | \_\_\_\_\_ | –1 |
| \((3\sqrt{5} - 2\sqrt{6})\) | \((3\sqrt{5} - 2\sqrt{6})(\_\_\_)\) | \_\_\_\_\_ | 21 |

The factors in the second column above are called **conjugate pairs**. How can you determine conjugate pairs? Use the technique above to write the following expressions without radicals in the denominator.

| \(\frac{2}{2 + \sqrt{7}}\) | \(\frac{2}{2 + \sqrt{7}} \cdot (\_\_\_)\) | \_\_\_\_\_ |
| \(\frac{3 + \sqrt{5}}{\sqrt{7} - \sqrt{10}}\) | \(\frac{3 + \sqrt{5}}{\sqrt{7} - \sqrt{10}} \cdot (\_\_\_)\) | \_\_\_\_\_ |
| \(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{11} + \sqrt{3}}\) | \(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{11} + \sqrt{3}} \cdot (\_\_\_)\) | \_\_\_\_\_ |
| \(\frac{-7\sqrt{2} + 4\sqrt{3}}{3\sqrt{2} - 5\sqrt{3}}\) | \(\frac{-7\sqrt{2} + 4\sqrt{3}}{3\sqrt{2} - 5\sqrt{3}} \cdot (\_\_\_)\) | \_\_\_\_\_ |

**Questions:**
1. How can we use conjugate pairs to rationalize the denominator?
2. How do you identify the conjugate pair?
3. What mathematical concepts are necessary to rationalize radicals?

Let us consolidate the results.
The previous activity required you to determine conjugate pairs. When do we use this skill?

c) **To divide radicals with a denominator consisting of at least two terms**, rationalize the denominator using its conjugate.

Examples:

\[
\frac{3}{\sqrt{3} - \sqrt{2}} = \frac{3}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{3\sqrt{3} + 3\sqrt{2}}{\sqrt{9} - \sqrt{4}} = \frac{3\sqrt{3} + 3\sqrt{2}}{3 - 2} = 3\sqrt{3} + 3\sqrt{2}
\]

\[
\frac{\sqrt{5} + 3}{\sqrt{5} - 3} = \frac{\sqrt{5} + 3}{\sqrt{5} - 3} \cdot \frac{\sqrt{5} + 3}{\sqrt{5} + 3} = \frac{\sqrt{25} + 3\sqrt{5} + 3\sqrt{5} + 9}{\sqrt{25} - 9} = \frac{5 + 3\sqrt{5} + 3\sqrt{5} + 9}{5 - 9} = \frac{14 + 6\sqrt{5}}{-4} = \frac{-7 - 3\sqrt{5}}{2}
\]

Since you already know how to divide radicals, sharpen that skill through answering the succeeding activities.

➤ **Activity 12: I’ll Let You Divide!**

Perform division of radicals and simplify the following expressions.

1. \(\sqrt{10} + \sqrt{2}\)
2. \(\sqrt{3} + \sqrt{3}\)
3. \(\sqrt{3} + \sqrt{3}\)
4. \(\sqrt{6} + \sqrt{6}\)
5. \(\sqrt{5} + \sqrt{7}\)
6. \(\sqrt{36} + \sqrt{6}\)
7. \(\sqrt{9} + \sqrt{3}\)
8. \(\sqrt{2} + \sqrt{2}\)
9. \(\sqrt{32a} + \sqrt{2}\)
10. \(\frac{1}{2 + \sqrt{5}}\)
11. \(\frac{1}{2 + \sqrt{5}}\)
12. \(\frac{1}{3 - \sqrt{11}}\)
13. \(\frac{1}{\sqrt{7} - \sqrt{5}}\)
14. \(\frac{1}{\sqrt{12} + \sqrt{7}}\)
15. \(\frac{1}{\sqrt{6} - \sqrt{3}}\)

Source (Modified): EASE Modules, Year 2-Module 5 Radical Expressions page 17

How well have you answered the previous activity? Keep in mind the important concepts/skills in dividing radicals. Test your understanding by answering the next activity.

➤ **Activity 13: Justify Your Answer**

Identify if the given process below is TRUE or FALSE based on the division of radicals then state your reason. For those you identified as false, make them true by writing the correct part of the solution.

1. \(\sqrt{10} + \sqrt{2}\)
2. \(\sqrt{3} + \sqrt{3}\)
3. \(\sqrt{3} + \sqrt{3}\)
4. \(\sqrt{6} + \sqrt{6}\)
5. \(\sqrt{5} + \sqrt{7}\)
6. \(\sqrt{36} + \sqrt{6}\)
7. \(\sqrt{9} + \sqrt{3}\)
8. \(\sqrt{2} + \sqrt{2}\)
9. \(\sqrt{32a} + \sqrt{2}\)
10. \(\frac{1}{2 + \sqrt{5}}\)
11. \(\frac{1}{2 + \sqrt{5}}\)
12. \(\frac{1}{3 - \sqrt{11}}\)
13. \(\frac{1}{\sqrt{7} - \sqrt{5}}\)
14. \(\frac{1}{\sqrt{12} + \sqrt{7}}\)
15. \(\frac{1}{\sqrt{6} - \sqrt{3}}\)
<table>
<thead>
<tr>
<th>True or False</th>
<th>Why?</th>
<th>If false, write the correct part of the solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplify $\sqrt{3xy^2}$, where $x &gt; 0$ and $y &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\sqrt{3xy^2}}{\sqrt{2x^2y}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{3xy^2}{(2x^2y)^{\frac{1}{2}}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{(3xy^2)^{\frac{1}{2}}}{(2x^2y)^{\frac{1}{2}}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{\sqrt{9x^2y^4}}{2x^{2}y}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{\sqrt{9y^{4}}}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{i\sqrt{9y^{3}}}{8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{i\sqrt{72y^{3}}}{16}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\sqrt{3xy^2}}{\sqrt{2x^2y}} = \frac{i\sqrt{72y^{3}}}{2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>True or False</th>
<th>Why?</th>
<th>If false, write the correct part of the solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplify $\frac{\sqrt{5}}{\sqrt{8}}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{\sqrt{5}}{2 \cdot \sqrt{2} \cdot \sqrt{5}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{\sqrt{25}}{2\sqrt{10}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{5}{2\sqrt{10}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Questions:

1. How do you think the given expressions were simplified? What processes did you observe?
2. How do we divide radicals with the same indices?
3. How do we divide radicals with different indices?
4. How do we simplify expressions with binomial radicals in the denominator?
5. What important understanding is necessary to simplify the given expression?

In the previous activity, you were able to identify whether the given process is correct or not based on valid mathematical facts or reason. Moreover, you were able to write the correct process in place of the incorrect one. The preceding activity aided you to further develop your skill in simplifying radicals.

Since you are now capable of simplifying expressions with radicals in the denominator, formulate your own conclusion through answering the next activity.

---

### Activity 14: Generalization

Write your generalization regarding simplifying radicals in your notebooks.

*In division of radicals…*
Activity 15: A Noisy Game!

Perform the indicated operations. Then, fill up the next table with the letter that corresponds to the correct answer.

Why is tennis a noisy game?

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>$\sqrt{2} \div \sqrt{3}$</td>
<td>P</td>
<td>$\sqrt[3]{8} \div \sqrt[3]{6}$</td>
<td>R</td>
</tr>
<tr>
<td>I</td>
<td>$\sqrt[4]{4} \div \sqrt[4]{6}$</td>
<td>E</td>
<td>$7 \div \sqrt[6]{6} + \sqrt[5]{5}$</td>
<td>K</td>
</tr>
<tr>
<td>L</td>
<td>$\sqrt{2} \div \sqrt{2}$</td>
<td>C</td>
<td>$\sqrt{5} \div \sqrt{15}$</td>
<td>Y</td>
</tr>
<tr>
<td>S</td>
<td>$\sqrt{2} \div (2 + \sqrt{3})$</td>
<td>V</td>
<td>$\frac{1}{\sqrt{x}}$</td>
<td>E</td>
</tr>
<tr>
<td>A</td>
<td>$5\sqrt{63} \div 6\sqrt{7}$</td>
<td>A</td>
<td>$\sqrt[20]{400}$</td>
<td>R</td>
</tr>
<tr>
<td>E</td>
<td>$\frac{6\sqrt{28}}{3\sqrt{4}}$</td>
<td>R</td>
<td>$\sqrt{80} \div \sqrt{5}$</td>
<td>A</td>
</tr>
<tr>
<td>R</td>
<td>$\frac{1}{2 + \sqrt{5}}$</td>
<td>Y</td>
<td>$10\sqrt{18} \div 2\sqrt{9}$</td>
<td>T</td>
</tr>
<tr>
<td>A</td>
<td>$\frac{3}{\sqrt{3} - 1}$</td>
<td>E</td>
<td>$\sqrt{25} \div \sqrt{625}$</td>
<td>S</td>
</tr>
</tbody>
</table>

| $\frac{\sqrt{6}}{3}$ | $\frac{\sqrt{x}}{x}$ | $7\sqrt{6} - 7\sqrt{5}$ | 4 | $5\sqrt{2}$ | $\frac{\sqrt[12]{12}}{3}$ | $\sqrt{2}$ | $\frac{5}{2}$ |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\frac{\sqrt{15bxy}}{5y}$ | $\frac{1}{5}$ | $\sqrt{5}$ | $\sqrt{6}$ | $\frac{3\sqrt{3} + 3}{2}$ | $\frac{\sqrt{18}}{3}$ | $2\sqrt{2} - \sqrt{6}$ | $2\sqrt{7}$ |
| $\frac{\sqrt{75}}{5}$ | $4\sqrt{2}$ | $-2 + \sqrt{5}$ | $2\sqrt{5}$ | $\frac{\sqrt{3}}{3}$ | $4\sqrt{2} - 3\sqrt{7}$ | $3\sqrt{2}$ | $\frac{25}{6}$ |

Source (Modified): EASE Modules, Year 2 – Module 5 Radical Expressions, page 18
Questions:
1. What important concepts/processes did you use in simplifying radicals?
3. How can you apply this skill to real-life situations?
4. Have you encountered any difficulties while solving? If yes, what are your plans to overcome them?

You just tried your understanding of the topic by answering the series of activities given to you in the previous section. Let us now try to deepen that understanding in the next section.

What to REFLECT and UNDERSTAND

Your goal in this section is to take a closer look at some aspects of the topic. You are now ready to answer the exercises given in this section. The activities aim to intensify the application of the different concepts you have learned.

➤ Activity 16: Transformers III

Transform and simplify each radical form into exponential form and vice versa. Then, answer the follow-up questions.

Questions:
1. What are your answers?
2. How did you arrive at your answers?
3. Are there concepts/processes to strictly follow in writing expressions with rational exponents to radicals?
4. Are there concepts/processes to strictly follow in writing radicals as expressions with rational exponents?
5. How can you apply the skills/concepts that you learned on exponents in a real-life situation?
In the previous activity, you were able to apply your understanding of expressions with rational exponents and radicals in simplifying complicated expressions. Did you perform well in the preceding activity? How did you do it? The next activity will deal with the formulating the general rule of operating radicals.

➤ Activity 17: Therefore I Conclude That…!

Answer the given activity by writing the concept/process/law used in simplifying the given expression, where each variable represents a positive real number.

<table>
<thead>
<tr>
<th></th>
<th>WHY?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$4\sqrt[4]{4} + 5\sqrt[4]{4}$</td>
</tr>
<tr>
<td></td>
<td>$9\sqrt[4]{4}$</td>
</tr>
<tr>
<td>2.</td>
<td>$3\sqrt{b} + 4\sqrt{b}$</td>
</tr>
<tr>
<td></td>
<td>$a\sqrt{b} + c\sqrt{b}$</td>
</tr>
<tr>
<td></td>
<td>$7\sqrt{b}$</td>
</tr>
<tr>
<td>3.</td>
<td>$a\sqrt{b} + c\sqrt{b}$</td>
</tr>
</tbody>
</table>

My conclusion:

<table>
<thead>
<tr>
<th></th>
<th>WHY?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\frac{\frac{1}{2} \cdot \frac{1}{2}}{5 \cdot 6}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{30}$</td>
</tr>
<tr>
<td></td>
<td>$x^{\frac{1}{2}} \cdot y^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>2.</td>
<td>$(x \cdot y)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{xy^{\frac{1}{2}}}{\sqrt{xy}}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{\frac{1}{n} \cdot \frac{1}{n}}{x^{\frac{1}{n}} \cdot y^{\frac{1}{n}}}$</td>
</tr>
<tr>
<td></td>
<td>$\sqrt[2n]{xy}$</td>
</tr>
</tbody>
</table>

My conclusion:
Questions:
1. How did you arrive at your conclusion?
2. What important insights have you gained from the activity?
3. Choose from the remaining lessons in radicals and do the same process on arriving at your own conclusion.

You were able to formulate your own conclusion on how to simplify radicals through the previous activity. The next activity will deal with the application of radicals to real-life related problems.

➤ Activity 18: Try to Answer My Questions!

Read carefully the given problem then answer the questions that follow.

If each side of a square garden is increased by 4 m, its area becomes 144 m².
1. What is the measure of its side after increasing it?
2. What is the length of the side of the original square garden?
3. Supposing the area of a square garden is 192 m², find the length of its side.

A square stock room is extended at the back in order to accommodate exactly the cartons of canned goods with a total volume of 588 m³. If the extension can exactly accommodate 245 m³ stocks, then find the original length of the stock room.
1. What are the dimensions of the new stock room?
2. Assuming that the floor area of a square stock room is 588 m², determine the length of its side.
3. Between which consecutive whole numbers can we find this length?

A farmer is tilling a square field with an area of 900 m². After 3 hrs, he tilled of the given area.
1. Find the side of the square field.
2. What are the dimensions of the tilled portion?
3. If the area of the square field measures 180 m², find the length of its side?
4. Between which consecutive whole numbers can we find this length?

A square swimming pool having an area of 25 m² can be fully filled with water of about 125 m³.
1. What are the dimensions of the pool?
2. If only of the swimming pool is filled with water, how deep is it?
3. Suppose the area of the square pool is 36 m², find the length of its side.

Source: BEAM Learning Guide, Year 2– Mathematics, Module 10: Radical Expressions in General, Mathematics 8 Radical Expressions; pages 41–44
The previous activity provides you with an opportunity to apply your understanding of simplifying radicals in solving real-life problems.
Try the next activity where you will test your skill of developing your own problem.

➤ Activity 19: Base It on Me!
Formulate a problem based on the given illustration then answer the questions that follow.

Approximately, the distance $d$ in kilometers that a person can see to the horizon is represented by the equation $d = \sqrt{\frac{3h}{2}}$, where $h$ is height from the person.

Questions:
1. How would you interpret the illustration based on the given formula?
2. What problem did you formulate?
3. How can you solve that problem?
4. How can you apply the skills/concepts that you learned from this activity in real-life situations?
Approximately, time $t$ in seconds that it takes a body to fall a distance $d$ in meters is represented by the equation $t = \frac{3d}{\sqrt{g}}$, where $g$ is the acceleration that is due to gravity equivalent to 9.8 m/s$^2$.

Questions:
1. How would you interpret the illustration based on the given formula?
2. What problem did you formulate?
3. How can you solve that problem?
4. How can you apply the skills/concepts that you learned on this activity in real-life situations?

How did you come up with your own problem based on the illustration? Have you formulated and solved it correctly? If not, try to find some assistance, for the next activity will still deal with formulating and solving problems.

➤ Activity 20: What Is My Problem?
Develop a problem based on the given illustration below.

Questions:
1. How would you interpret the illustration?
2. What problem have you formulated?
3. How can you solve that problem?
4. How can you apply the skills/concepts that you learned on this activity in real-life situation?

How do you feel when you can formulate and solve problems that involve radicals? Let me know the answer to that question by filling-out the next activity.
Activity 21: IRF Sheet (Revisited)

Below is an IRF Sheet. It will help check your understanding of the topics in this lesson. You will be asked to fill in the information in different sections of this lesson. This time, kindly fill in the second column that deals with your revised ideas.

<table>
<thead>
<tr>
<th>INITIAL</th>
<th>REVISE</th>
<th>FINAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are your initial ideas about radicals?</td>
<td>What are your new ideas?</td>
<td>Do not answer this part yet</td>
</tr>
<tr>
<td>With answer already</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now that you know how to simplify radicals, let us now solve real-life problems involving this understanding.

What to TRANSFER:

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding of the lesson. This task challenges you to apply what you learned about simplifying radicals. Your work will be graded in accordance with the rubric presented.

Activity 22: Transfer Task

You are an architect in a well-known establishment. You were tasked by the CEO to give a proposal on the diameter of the establishment’s water tank design. The tank should hold a minimum of 950 m³. You were required to have a proposal presented to the Board. The Board would like to see the concept used, its practicality, accuracy of computation, and the organization of the report.
<table>
<thead>
<tr>
<th>Categories</th>
<th>4 Excellent</th>
<th>3 Satisfactory</th>
<th>2 Developing</th>
<th>1 Beginning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Concept</td>
<td>Demonstrate a thorough understanding of the topic and use it appropriately to solve the problem</td>
<td>Demonstrate a satisfactory understanding of the concepts and use it to simplify the problem</td>
<td>Demonstrate incomplete understanding and have some misconceptions</td>
<td>Shows lack of understanding and has severe misconceptions</td>
</tr>
<tr>
<td>Accuracy of Computation</td>
<td>All computations are correct and are logically presented.</td>
<td>The computations are correct.</td>
<td>Generally, most of the computations are not correct.</td>
<td>Errors in computations are severe.</td>
</tr>
<tr>
<td>Practicality</td>
<td>The output is suited to the needs of the client and can be executed easily. Ideas presented are appropriate to solve the problem.</td>
<td>The output is suited to the needs of the client and can be executed easily.</td>
<td>The output is suited to the needs of the client and cannot be executed easily.</td>
<td>The output is not suited to the needs of the client and cannot be executed easily.</td>
</tr>
<tr>
<td>Organization of the Report</td>
<td>Highly organized, flows smoothly, observes logical connections of points</td>
<td>Satisfactorily organized. Sentence flow is generally smooth and logical</td>
<td>Somewhat cluttered. Flow is not consistently smooth, appears disjointed</td>
<td>Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.</td>
</tr>
</tbody>
</table>

Were you able to accomplish the task properly? How was the process/experience in doing it? Was it challenging yet an exciting task? Let us summarize that experience by answering the IRF sheet and synthesis journal on the next page.
Activity 23: IRF Sheet (finalization)

Below is an IRF Sheet. It will help check your understanding of the topics in this lesson. You will be asked to fill in the information in different sections of this lesson. This time, kindly fill in the third column that deals with your final ideas about the lesson.

<table>
<thead>
<tr>
<th>INITIAL</th>
<th>REVISE</th>
<th>FINAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are your initial ideas about radicals?</td>
<td>What are your new ideas?</td>
<td>What are your final ideas about the lesson?</td>
</tr>
<tr>
<td>With answer already</td>
<td>With answer already</td>
<td></td>
</tr>
</tbody>
</table>

Activity 23: Synthesis Journal

Complete the table below by answering the questions.

<table>
<thead>
<tr>
<th>How do I find the performance task?</th>
<th>What are the values I learned from the performance task?</th>
<th>How do I learn them? What made the task successful?</th>
<th>How will I use these learning/insights in my daily life?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summary/Synthesis/Generalization:

This lesson was about writing expressions with rational exponents to radicals and vice versa, simplifying and performing operations on radicals. The lesson provided you with opportunities to perform operations and simplify radical expressions. You identified and described the process of simplifying these expressions. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the next lesson on radicals.
What to KNOW

How can we apply our understanding of simplifying radicals to solving radical equations? Why do we need to know how to solve radical equations? Are radicals really needed in life outside mathematics studies? How can you simplify radicals? How can the understanding of radicals help us solve problems in daily life?

In this lesson we will address these questions and look at some important real-life applications of radicals.

➤ Activity 1: Let's Recall!

Solve the given problem below.

Approximately, the distance $d$ in miles that a person can see to the horizon is represented by the equation $d = \sqrt{\frac{3h}{2}}$, where $h$ is the height where the person is. How far can a man see if he is 5 meters above the ground? (1 mile = 1,609.3 m)

Questions:
1. How far can a man see if he is 5 meters above the ground?
2. How did you solve the problem? What concepts/skills have you applied?
3. What is your mathematical representation of the problem?
4. What do you think might happen if we replace the radicand with a variable? Will it still be possible to solve the problem?

A man walks 4 meters to the east going to school and then walks 9 meters northward going to the church.

Questions:
1. How far is he from the starting point which is his house?
2. How did you arrive at the answer to the problem?
3. What important concepts/skills have you applied to arrive at your answer?
4. Can you think of an original way to solve the problem?

How did you answer the activity? Did you recall the skills that you learned from the previous topic? Are you now more comfortable with radicals? Let me know your initial ideas by answering the next activity.
Activity 2: K–W–L Chart
Fill-in the chart below by writing what you Know and what you Want to know about the topic “solving radical equations.”

<table>
<thead>
<tr>
<th>What I Know</th>
<th>What I Want to Know</th>
<th>What I Learned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Do not answer this part yet</td>
</tr>
</tbody>
</table>

In the preceding activity you were able to cite what you know and what you want to know about this lesson. Try to answer the next activity for you to have an overview of the lesson’s application.

Were you able to answer the previous activity? How did you do it? Find out how to correctly answer this problem as we move along with the lesson. Recall all the properties, postulates, theorems, and definitions that you learned from geometry because it is needed to answer the next activity.

Activity 3: Just Give Me a Reason!
Answer the given activity by writing the concept/process/law used to simplify the given equation.

1. $\sqrt{x + 2} = 3$
   Why?
   $$(x + 2)^{\frac{1}{2}} = 3$$
   $$\left(\left[x + 2\right]^\frac{1}{2}\right)^3 = 3^3$$
   $$(x + 2)^{\frac{3}{2}} = 3^3$$
   $$(x + 2) = 3^3$$
   $$x + 2 = 27$$
   $$x = 27 - 2$$
   $$x = 25$$

Checking:
$$\sqrt{x + 2} = ?$$
$$\sqrt{25 + 2} = ?$$
$$\sqrt{27} = ?$$
$$3 = ?$$

Conclusion:

2. $-3\sqrt{x + 2} = x - 16$
   Why?
   $$-3(x + 2)^{\frac{1}{2}} = x - 16$$
   $$\left[-3(x + 2)^{\frac{1}{2}}\right]^2 = (x - 16)^2$$
   $$9(x + 2) = (x - 16)^2$$
   $$9x + 18 = x^2 - 32x + 256$$
   $$x^2 - 41x + 238 = 0$$
   $$x = \frac{41 \pm \sqrt{729}}{2}$$
   $$x = 7, x = 34$$

Checking:
$$x = 7$$
$$-3\sqrt{x + 2} = ?$$
$$-3\sqrt{7} + 2 = ?$$
$$-3\sqrt{9} = ?$$
$$-3 \neq 9$$
$$-9 \neq 9$$
$$-18 \neq 18$$

Conclusion:
3. \( \sqrt{x} = 8 \), where \( x > 0 \)

\[
\begin{align*}
\frac{1}{x^{\frac{1}{2}}} &= 8 \\
\left( x^{\frac{1}{2}} \right)^2 &= (8)^2 \\
x &= 64
\end{align*}
\]

Checking:

\[
\sqrt{x} = 8 \\
8 = 8
\]

Conclusion:

Questions:

1. How did you arrive at your conclusion?
2. How would you justify your conclusions? What data was used?
3. Can you elaborate on the reason at arriving at the conclusion?
4. Can you find an alternative process of solving this type of problem?
5. In the second problem, 34 is called an extraneous root. How do you define an extraneous root?
6. Compare your conclusions and reasons with that of your classmates'. What have you observed? Have you arrived at the same answers? Why?
7. What important insights have you learned from the activity?

How did you find the preceding activities? You were able to formulate conclusions based on the reasons for the simplifying process. You learned that when solving radical expressions, squaring both sides of an equation may sometimes yield an extraneous root. But how are radicals used in solving real-life problems? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on the topic.
Carefully analyze the given examples below then answer the questions that follow.

\[
\begin{align*}
\sqrt{x} - 2 &= 10 \\
\sqrt{x} &= 10 + 2 \\
\sqrt{x} &= 12 \\
\frac{1}{\sqrt{x}} &= 12 \\
\left(\frac{1}{\sqrt{x}}\right)^2 &= (12)^2 \\
x^{\frac{1}{3}} &= 12^\frac{1}{3} \\
x &= 144
\end{align*}
\]

Checking:
\[
\begin{align*}
\sqrt{x} - 2 &= 10 \\
\sqrt{144} - 2 &\neq 10 \\
12 - 2 &\neq 10 \\
10 &\neq 10
\end{align*}
\]

\[
\begin{align*}
3\sqrt{x+1} &= 2 \\
3(x+1)^{\frac{1}{3}} &= 2 \\
\left[3(x+1)^{\frac{1}{3}}\right]^3 &= 2^3 \\
3\left(x+1\right)^{\frac{3}{3}} &= 2^3 \\
27(x+1) &= 8 \\
\frac{27(x+1)}{27} &= \frac{8}{27} \\
x + 1 &= \frac{8}{27} \\
x &= \frac{8}{27} - 1 \\
x &= -\frac{19}{27}
\end{align*}
\]

Checking:
\[
\begin{align*}
\sqrt[3]{2x+1} &= \sqrt[3]{x+8} \\
(2x+1)^{\frac{1}{3}} &= (x+8)^{\frac{1}{3}} \\
\left[(2x+1)^{\frac{1}{3}}\right]^3 &= \left[(x+8)^{\frac{1}{3}}\right]^3 \\
(2x+1) &= (x+8)^3 \\
2x + 1 &= x + 8 \\
2x - x &= 8 - 1 \\
x &= 7
\end{align*}
\]

Checking:
\[
\begin{align*}
\sqrt[3]{\sqrt{2x+1}} &= \sqrt[3]{\sqrt{x+8}} \\
\sqrt[6]{2x+1} + 1 &\neq \sqrt[6]{x+8} \\
\sqrt[6]{2(7)+1} &\neq \sqrt[6]{7+8} \\
\sqrt[6]{15} &\neq \sqrt[6]{15}
\end{align*}
\]

Questions:
1. Based on the illustrative examples, how would you define radical equations?
2. How were the radical equations solved?
3. Can you identify the different parts of the solution and the reason/s behind each?
4. What important concepts/skills were needed to solve radical equations?
5. What judgment can you make on how to solve radical equations?
Let us consolidate the results.

A **radical equation** is an equation in which the variable appears in a radicand.

Examples of radical equations are:

- \( a) \sqrt{x} = 7 \)
- \( b) \sqrt{x + 2} = 3 \)
- \( c) \sqrt{2x - 3} = \sqrt{x} + 5 \)

In solving radical equations, we can use the fact that if two numbers are equal, then their squares are equal. In symbols; \( a = b \), then \( a^2 = b^2 \).

Examples:

If \( \sqrt{9} = 3 \) are equal, then \( \left( \sqrt{9} \right)^2 = (3)^2 \) are equal.

As a result \( \left( 9 \frac{1}{2} \right)^2 = (3)^2 \).

\[
\begin{align*}
9 \frac{1}{2} &= 9 \\
9 &= 9 
\end{align*}
\]

Checking:

\[
\begin{align*}
\sqrt{x} &= 9 \\
x &= 9 \\
\end{align*}
\]

This is the only solution

Extraneous Root

Analyze the illustrative examples below then try to define an **extraneous root**.

\[
\begin{align*}
x - 6 &= \sqrt{x} \\
x - 6 &= (x)^{\frac{1}{2}} \\
(x - 6)^2 &= \left( (x)^{\frac{1}{2}} \right)^2 \\
(x - 6)^2 &= x^{\frac{1}{2}} \\
x^2 - 12x + 36 &= x \\
x^2 - 12x - x + 36 &= 0 \\
x^2 - 13x + 36 &= 0 \\
(x - 9)(x - 4) &= 0 \\
x &= 9, x = 4 \\
\end{align*}
\]

Checking:

\[
\begin{align*}
x &= 9 \\
x - 6 &= \sqrt{x} \\
9 - 6 &= \sqrt{9} \\
3 &= 3 \\
\end{align*}
\]

This is the only solution

Extraneous Root

\[
\begin{align*}
x &= 4 \\
x - 6 &= \sqrt{x} \\
4 - 6 &= \sqrt{4} \\
-2 &\neq 2 \\
\end{align*}
\]
Questions:
1. How would you define an extraneous root based on the illustrative examples?
2. What data have you used to define an extraneous root?
3. How is the process of checking related to finalizing your answer to a problem?
4. What insights have you gained from this discussion?

Let us consolidate the results.

Important: If the squares of two numbers are equal, the numbers may or may not be equal. Such as, \((-3)^2 = 3^2\), but \(-3 \neq 3\). It is therefore important to check any possible solutions for radical equations. Because in squaring both sides of a radical equation, it is possible to get extraneous solutions.

To solve a radical equation:
1. Arrange the terms of the equation so that one term with radical is by itself on one side of the equation.
2. Square both sides of the radical equation.
3. Combine like terms.
4. If a radical still remains, repeat steps 1 to 3.
5. Solve for the variable.
6. Check apparent solutions in the original equation.

You are now knowledgeable on how to solve radical equations. Let us try to apply that skill in solving problems.
Carefully analyze the given examples below then answer the questions that follow.

A certain number is the same as the cube root of 16 times the number. What is the number?

Representation:
Let \( m \) be the number

Mathematical Equation:
\[
 m = \sqrt[3]{16m}
\]

Solution:
\[
m = \sqrt[3]{16m}
\]
\[
m = (16m)^{\frac{1}{3}}
\]
\[
(m)^3 = (16m)^{\frac{3}{3}}
\]
\[
m^3 = (16m)^{\frac{3}{3}}
\]
\[
m^3 = 16m
\]
\[
m = 16m
\]
\[
m^3 - 16m = 0
\]
\[
m(m^2 - 16) = 0
\]
\[
m^2 - 16 = (m + 4)(m - 4)
\]
\[
m = 0, m = -4, m = 4
\]

Final answer: The numbers are 0, -4, and 4.

Questions:
1. How were the radical equations solved?
2. What are the different parts of the solution and the reason/s behind it?
3. What important concepts/skills were needed to solve radical equations?
4. What judgment can you make on how to solve radical equations?
5. How do you solve real-life related problems involving radicals?
A woman bikes 5 kilometers to the east going to school and then walks 9 kilometers northward going to the church. How far is she from the starting point which is her house?

We can illustrate the problem for better understanding. Since the illustration forms a right triangle, therefore we can apply $c = \sqrt{a^2 + b^2}$ to solve this problem.

Let: $a = 9$ m  
$b = 5$ m

Solution:

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
c &= \sqrt{a^2 + b^2} \\
c &= \sqrt{(9m)^2 + (5m)^2} \\
c &= \sqrt{81m^2 + 25m^2} \\
c &= \sqrt{106m^2} \\
c &= 106m
\end{align*}
\]

Checking:

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
(\sqrt{106m})^2 &= (9m)^2 + (5m)^2 \\
106m^2 &= 81m^2 + 25m^2 \\
106m^2 &= 106m^2 \\
c &= \sqrt{81m^2 + 25m^2}
\end{align*}
\]

Final Answer: The woman is $\sqrt{106}$ m far from her house or approximately between 10 m and 11 m.

Now that you already know how to solve radical equations and somehow relate that skill to solving real-life problems, let us try to apply this understanding by answering the following activities.
What to PROCESS

Your goal in this section is to apply your understanding to solving radical equations. Towards the end of this module, you will be encouraged to apply your understanding on radicals to solving real-life problems.

➤ Activity 4: Solve Me!

Solve the following radical equations and box the final answer.

1. $\sqrt{x} = 10$
2. $\sqrt{2m} = 4$
3. $-5\sqrt{b} = -50$
4. $\sqrt{n + 2} = 3$
5. $\sqrt{2s + 10} = 4$
6. $\sqrt{x - 1} = x - 7$
7. $\sqrt{x - 3} + \sqrt{x} = 3$
8. $\sqrt{3a + 9} = \sqrt{6a + 15}$
9. $4\sqrt{5m - 20} = 16$
10. $2\sqrt{h + 5} = 4\sqrt{2h - 15}$

Questions:
1. What are your solutions to the given radical equations?
2. How did you solve the given equations using what you learned in radicals?
3. Find a partner and try to compare your answers,
   • How many of your answers are the same?
   • How many are different?
4. Compare the solution of those problems with different answers and come up with the correct one.
5. Have you encountered any difficulties in solving radical equations? If yes, what are your plans to overcome these?

You just tried your skill in solving radical equations in the previous activity. How did you perform? Did you answer majority of the equations correctly? The previous activity dealt with solving radical equations. Try to solve the next activity that requires postulates, definitions, and theorems that you learned from geometry.
**Activity 5: The Reasons Behind My Actions!**

Solve the radical equations. Write your solution and the property, definition, or theorem that you used in your solution.

<table>
<thead>
<tr>
<th>Radical Equations</th>
<th>Solution</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6\sqrt{8a^2} - 72 = 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{b^2 + 8} = 4\sqrt{3b^2} + 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5\sqrt{5x} + 2 = 10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{5x + 10} = \sqrt{6x + 4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Questions:
1. How is your understanding of radicals related to answering this activity correctly?
2. What parts of the process are significant in arriving at the correct answer?
3. How could you solve problems without bases/reasons?
4. Have you encountered any difficulties in solving the problems? If yes, what are your plans to overcome them?
5. What important insights have you gained from this activity?
6. What judgment would you make regarding the relationship of the parts of the solutions and its respective reason?

In this section, you were able to solve radical equations and solve real-life problems involving radicals. One of the activities made you realize that using mathematical reasoning, you will arrive at the correct answer to solve the given problem. You just tried your understanding of the topic by answering the series of activities given to you in the previous section. Let us now try to sharpen these knowledge and skills in the next section.

What to REFLECT and UNDERSTAND

Your goal in this section is to take a closer look at some aspects of the topic. I hope that you are now ready to answer the exercises given in this section. Expectedly, the activities aim to intensify the application of the different concepts you have learned.

➤ Activity 6: Problem–Solved!

Solve the problems below by analyzing the given statements and answering the questions that follow.

A. Number problems.
   1. Five times the square root of 1 less than a number is equal to 3 more than the number. Find the number.
   2. What number or numbers are equal to their own square roots?
   3. The sum of a number and its square root is equal to 0. Find the number.
   4. Find the number such that twice its square root is 14.
   5. Find the number such that the square root of four more than five times the number is 8.
B. Approximately, the distance, $d$ in miles that a person can see to the horizon is represented by the equation $d = \sqrt{\frac{3h}{2}}$, where $h$ is the height where the person is. (1 mile = 1609.3 m)

1. How far can you see to the horizon through an airplane window at a height of 8000 m?
2. How far can a sailor see to the horizon from the top of a 20 m mast?
3. How far can you see to the horizon through an airplane window at a height of 9800 m?
4. How far can a sailor see from a top of a 24 m mast?

C. The formula $r = 2\sqrt{L}$ can be used to approximate the speed $r$, in miles per hour, of a car that has left a skid mark of $L$, in feet.

1. How far will a car skid at 50 mph? at 70 mph?
2. How far will a car skid at 60 mph? at 100 mph?

D. Carpenters stabilize wall frames with a diagonal brace. The length of the brace is given by $L = \sqrt{H^2 + W^2}$.

1. If the bottom of the brace is attached 9 m from the corner and the brace is 12 m long, how far up the corner post should it be nailed?

Source (Modified): EASE Modules, Year 2 – Module 6 Radical Expressions, pages 14–17

In the previous activity you were able to apply your understanding of solving radical equations to solving real-life problems that involve radicals. Let us put that understanding to the test by answering the next activity.

➤ Activity 7: More Problems Here!

Solve the given problems then answer the questions that follow.

Juan is going to Nene's house to do a school project. Instead of walking two perpendicular streets to his classmate’s house, Juan will cut a diagonal path through the city plaza. Juan is 13 meters away from Nene’s street. The distance from the intersection of the two streets to Nene’s house is 8 meters.

Questions:
1. How would you illustrate the problem?
2. How far will Juan travel along the shortcut?
3. How many meters will he save by taking the short cut rather than walking along the sidewalks?
4. If one of the distances increases/decreases, what might happen to the distance of the short-cut? Justify your answer.
5. What mathematical concepts did you use?
A wire is anchored on a 9-meter pole. One part is attached to the top of the pole and the other is 2 meters away from the base.

**Questions:**
1. How long is the wire?
2. What will happen if the wire is farther/nearer to the base? Justify your answer.
3. What mathematical concepts did you use?

If a 36-storey building is 110-meter high, using the formula \( d = \sqrt{\frac{3h}{2}} \) for sight distance where \( d \) is the distance in miles and \( h \) is height where the person is, how far can you see the building on a clear day? (1 mile = 1609.3 m)

**Questions:**
1. How would you illustrate the problem?
2. How far can you see the building on a clear day?
3. If the height of the building increases/decreases, what might happen to the sight distance? Justify your answer.

The previous activities gave you the opportunity to apply your understanding of solving radical equations to solving real-life problems that involve radicals. Try to answer the next activity where you are required to create and solve your own problem.

➤ **Activity 8: What Is My Problem!**
Formulate a problem based on the given illustration then answer the questions that follow.

![Illustration of a lighthouse and a ship with distances marked]

*Note: (1 nautical mile = 1852 meters)*

**Questions:**
1. How did you interpret the illustration?
2. What problem have you formulated?
3. How did you solve the problem? What concepts/skills have you applied?
4. Show your solution.
5. What is your final answer?
6. If the height of the light house changed from 63 meters to 85 meters, what will be its effect on the distance of the ship from the base of the light house?
7. How will you apply the concepts of radicals to a real-life situation?

\[ T = 2\pi \sqrt{\frac{L}{32}} \]

is the formula which gives the time (T) in seconds for a pendulum of length (L) in feet (ft) to complete one full cycle.

Questions:
1. How did you understand the illustration?
2. What problem have you formulated?
3. How did you solve the problem? What concepts/skills have you applied?
4. Show your solution.
5. What is your final answer?
6. How long is the pendulum if it will take 1 second to complete one full cycle?
7. How would you apply the concepts of radicals to a real-life situation?

\[ v = \sqrt{2gd} \]

the velocity of a free falling object can be determined by this equation, where \( v \) is measured in feet per second \( \left( \frac{\text{ft}}{\text{sec}} \right) \), \( g = 32 \) feet per second squared, \( \left( \frac{\text{ft}}{\text{sec}^2} \right) \), \( d \) is the distance in feet, (ft) the object has fallen.

Questions:
1. How did you understand the illustration?
2. What problem have you formulated?
3. How did you solve the problem? What concepts/skills have you applied?
4. Show your solution.
5. What is your final answer?
6. How would you apply the concepts of radicals to a real-life situation?

How did you find the previous activity? Does it stimulate your critical thinking? Have you formulated and solved the problem correctly?

The previous activity dealt with the application of radicals to real-life problems. Have you done well in answering this activity? Well then, I want to know what you have already learned by filling-out the next activity.

➤ Activity 9: K–W–L Chart

Fill-in the chart below by writing what you have learned from the topic “solving radical equations.”

<table>
<thead>
<tr>
<th>What I Know</th>
<th>What I Want to Know</th>
<th>What I Learned</th>
</tr>
</thead>
<tbody>
<tr>
<td>with answer already</td>
<td>with answer already</td>
<td></td>
</tr>
</tbody>
</table>

➤ Activity 10: Synthesis Journal

Fill-in the table below by answering the given question.

<table>
<thead>
<tr>
<th>Syntheses Journal</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>What interests me.</td>
<td>What I learned.</td>
<td>How can the knowledge of radical equations help us solve real-life problems?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now that you know well how to simplify radicals, let us now solve real-life problems involving this understanding.

What to TRANSFER

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

This task challenges you to apply what you learned about simplifying radicals. Your work will be graded in accordance with the rubric presented.
Activity 11: Transfer Task

Hang time is defined as the time that you are in the air when you jump. It can be calculated using the formula \( t = \frac{\sqrt{2h}}{g} \), where \( h \) is height in feet, \( t \) is time in seconds and \( g \) is the gravity given as \( \frac{32}{\text{sec}^2} \).

Your school newspaper is to release its edition for this month. As a writer/researcher of the sports column, you were tasked to create a feature regarding the hang time of your school’s basketball team members. Your output shall be presented to the newspaper adviser and chief editor and will be evaluated according to the mathematical concept used, organization of report, accuracy of computations, and practicality of your suggested game plan based on the result of your research.

Now that you are done with your work, use the rubric on the next page to check your work. Your work should show the traits listed under SATISFACTORY or 3. If your work has these traits, you are ready to submit your work.

If you want to do more, you work should show the traits listed under EXCELLENT or 4.

If your work does not have any traits under 3 or 4, revise your work before submitting it.

Rubrics for the Performance Task

<table>
<thead>
<tr>
<th>Categories</th>
<th>4 Excellent</th>
<th>3 Satisfactory</th>
<th>2 Developing</th>
<th>1 Beginning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Concept</td>
<td>Demonstrates a thorough understanding of the topic and uses it appropriately to solve the problem</td>
<td>Demonstrates a satisfactory understanding of the concepts and uses it to simplify the problem</td>
<td>Demonstrates incomplete understanding and has some misconceptions</td>
<td>Shows lack of understanding and have severe misconceptions</td>
</tr>
<tr>
<td>Accuracy of Computation</td>
<td>All computations are correct and are logically presented.</td>
<td>The computations are correct.</td>
<td>Generally, most of the computations are not correct.</td>
<td>Errors in computations are severe.</td>
</tr>
<tr>
<td>Practicality</td>
<td>The output is suited to the needs of the client and can be executed easily.</td>
<td>The output is suited to the needs of the client and can be executed easily.</td>
<td>The output is suited to the needs of the client and cannot be executed easily.</td>
<td>The output is not suited to the needs of the client and cannot be executed easily.</td>
</tr>
<tr>
<td>Organization of the Report</td>
<td>Highly organized, flows smoothly, and observes logical connections of points</td>
<td>Satisfactorily organized. Sentence flow is generally smooth and logical.</td>
<td>Somewhat cluttered. Flow is not consistently smooth, appears disjointed.</td>
<td>Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.</td>
</tr>
</tbody>
</table>

Were you able to accomplish the task properly? How was the process/experience in doing it? Was it a challenging yet an exciting task? Let us summarize that experience by answering the lesson closure.

➤ Activity 13: Summary
Complete the paragraph below.

Lesson Closure
This lesson ____________________________________________
_____________________________________________________
_____________________________________________________. One key idea is ____________________________
_____________________________________________________

This is important because ______________________________________
_____________________________________________________
______________________________________________________. Another key idea ____________________________
_____________________________________________________

In sum, this lesson ______________________________________
_____________________________________________________
_____________________________________________________
______________________________________________________
Summary/Synthesis/Generalization

This lesson was about solving radical equations. The lesson provided you with opportunities to solve radical equations and apply this understanding to a real-life situation. You identified and described the process of simplifying these expressions. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematical concepts and principles will facilitate your learning into the next lesson.

Glossary of Terms:

**Conjugate Pair** – two binomial radical expressions that have the same numbers but only differ in the sign that connects the binomials

**Dissimilar Radicals** – radicals with different order and having the same radicand or with same order and having a different radicand

**Exponent** – a number that says how many times the base is to be multiplied by itself

**Extraneous Solution** – a solution that does not satisfy the given equation

**Radical** – an expression in the form of \( \sqrt[n]{a} \), where \( n \) is a positive integer and \( a \) is an element of the real number system

**Radical equations** – equations containing radicals with variables in the radicand

**Rational Exponent** – an exponent in the form of \( \frac{m}{n} \), where \( m \) and \( n \) are integers and \( n \neq 0 \)

**Rationalization** – simplifying a radical expression by making the denominator free of radical

**Similar Radicals** – radicals with the same order and having the same radicand

References and Website Links Used in this Module:

**References:**


**References for Learner’s Activities:**

BEAM Learning Guide, Second Year – Mathematics, Module 10: Radicals Expressions in General, pages 31-33
Weblinks Links as References and for Learner’s Activities:

Applications of surface area. braining camp. http://www.brainingcamp.com/legacy/content/concepts/surface-area/problems.php
(Charge of electron) https://www.google.com.ph/#q=charge+of+electron
(Extraneous Solutions) http://www.mathwords.com/e/extraneous_solution.htm
(Formula for pendulum) http://hyperphysics.phy-astr.gsu.edu/hbase/pend.html
Gallon of Paint http://answers.ask.com/reference/other/how_much_does_one_gallon_of_paint_cover
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(Radio frequency)http://www.sengpielaudio.com/calculator-radiofrequency.htm
Small Number. Wikipediahttp://en.wikipedia.org/wiki/Small_number
Solving Radical Equations and Inequalities
(Speed of Light) http://www.space.com/15830-light-speed.html
(Square meter to square ft)http://www.metric-conversions.org/area/square-feet-to-square-meters.htm
( Square meter to square feet ) http://calculator-converter.com/converter_square_meters_to_square_feet_calculator.php
(Diameter of an atomic nucleus) http://www.princeton.edu/~achaney/tmve/wiki100k/docs/Atomic_nucleus.html