Module 2: Quadratic Functions

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We value your feedback and recommendations.

Department of Education
Republic of the Philippines
**MATHEMATICS GRADE 9**

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# Table of Contents

Module 2. **Quadratic Functions** ................................................................. 119  
  Module Map ............................................................................................ 121  
  Pre-Assessment .................................................................................... 122  
  Learning Goals and Targets ................................................................. 124  
  Lesson 1. Introduction to Quadratic Functions .................................. 125  
  Lesson 2. Graphs of Quadratic Functions .......................................... 140  
  Lesson 3. Finding the Equation of a Quadratic Function .................. 156  
  Lesson 4. Applications of Quadratic Functions ............................... 174  
  Glossary of Terms ............................................................................... 184  
  References and Website Links Used in this Module ...................... 184
I. INTRODUCTION AND FOCUS QUESTIONS

Have you ever asked yourself why PBA star players are good in free throws? How do angry bird expert players hit their targets? Do you know the secret key in playing this game? What is the maximum height reached by an object thrown vertically upward given a particular condition?

One of the most interesting topics in mathematics is the quadratic function. It has many applications and has played a fundamental role in solving many problems related to human life. In this module, you will be able to learn important concepts in quadratic functions which will enable you to answer the questions above. Moreover, you will also deal with the most common applications of quadratic functions.
II. LESSONS AND COVERAGE

This module consists of four lessons namely:

Lesson 1 – INTRODUCTION TO QUADRATIC FUNCTIONS
Lesson 2 – GRAPHS OF QUADRATIC FUNCTIONS
Lesson 3 – FINDING THE EQUATION OF A QUADRATIC FUNCTION
Lesson 4 – APPLICATIONS OF QUADRATIC FUNCTIONS

Objectives

In this module, you will learn to:

| Lesson 1 | • model real-life situations using quadratic functions  
|          | • differentiate quadratic functions from linear or other functions.  
|          | • represent and identify the quadratic function given  
|          | – table of values  
|          | – graphs  
|          | – equation  
|          | • transform the quadratic function in general form \( y = ax^2 + bx + c \) into standard form (vertex form) \( y = a(x - h)^2 + k \) and vice versa.  

| Lesson 2 | • draw the graph of the quadratic function  
|          | • given a quadratic function, determine the following: domain, range, intercepts, axis of symmetry, and the opening of the parabola.  
|          | • investigate and analyze the effects of changes in the variables \( a, h, \) and \( k \) in the graph of quadratic functions \( y = a(x - h)^2 + k \) and make generalizations.  
|          | • apply the concepts learned in solving real-life problems.  

| Lesson 3 | • determine the zeros of quadratic functions  
|          | • derive the equation of the quadratic function given  
|          | – table of values  
|          | – graphs  
|          | – zeros  
|          | • apply the concepts learned in solving real-life problems.  

| Lesson 4 | • solve problems involving quadratic functions  

120
III. PRE-ASSESSMENT

Part I

Find out how much you already know about this module. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Which of the following equations represents a quadratic function?
   a. \( y = 3 + 2x^2 \)
   b. \( 2y^2 + 3 = x \)
   c. \( y = 3x - 2^2 \)
   d. \( y = 2^x - 3 \)

2. The quadratic function \( f(x) = x^2 + 2x - 1 \) is expressed in standard form as
   a. \( f(x) = (x + 1)^2 + 1 \)
   b. \( f(x) = (x + 1)^2 - 2 \)
   c. \( f(x) = (x + 1)^2 + 2 \)
   d. \( f(x) = (x + 1)^2 - 1 \)

3. What is \( f(x) = -3(x + 2)^2 + 2 \) when written in the form \( f(x) = ax^2 + bx + c \)?
   a. \( f(x) = -3x^2 + 12x - 10 \)
   b. \( f(x) = 3x^2 - 12x + 10 \)
   c. \( f(x) = -3x^2 + 12x + 10 \)
   d. \( f(x) = -3x^2 - 12x - 10 \)

4. The zeros of the quadratic function described by the graph below is
   a. 1, 3
   b. -1, 3
   c. 1, -3
   d. -1, -3

5. The graph of \( y = x^2 - 3 \) is obtained by sliding the graph of \( y = x^2 \)
   a. 3 units downward
   b. 3 units upward
   c. 3 units to the right
   d. 3 units to the left

6. The quadratic function \( y = -2x^2 + 4x - 3 \) has
   a. real and unequal zeros
   b. real and equal zeros
   c. no real zeros
   d. equal and not real

7. What is an equation of a quadratic function whose zeros are twice the zeros of \( y = 2x^2 - x - 10 \)?
   a. \( f(x) = 2x^2 - 20x + 20 \)
   b. \( f(x) = x^2 - x - 20 \)
   c. \( f(x) = 2x^2 - 2x - 5 \)
   d. \( f(x) = 2x^2 - 2x - 10 \)
8. Which of the following shows the graph of \( f(x) = 2(x-1)^2 - 3 \)

![Graphs a, b, c, d](image)

9. Richard predicted that the number of mango trees, \( x \), planted in a farm could yield \( y = -20x^2 + 2800x \) mangoes per year. How many trees should be planted to produce the maximum number of mangoes per year?

a. 60  

b. 70  

c. 80  

d. 90

10. The path of an object when it is thrown can be modeled by \( S(t) = -16t^2 + 8t + 4 \) where \( S \) in feet is the height of the object \( t \) seconds after it is released. What is the maximum height reached by the object?

a. 3 ft  

b. 4 ft  

c. 5 ft  

d. 6 ft

11. CJ wrote a function of the path of the stone kicked by Lanlan from the ground. If the equation of the function he wrote is \( S(t) = 16t^2 + 8t + 1 \), where \( S \) is the height of stone in terms of \( t \), the number of seconds after Lanlan kicks the stone. Which of the statement is true?

a. CJ’s equation is not correct.

b. CJ’s equation described the maximum point reached by the stone.

c. The equation is possible to the path of the stone.

d. The equation corresponds to the path of the stone.

12. An object is fired straight up with a velocity of 64 ft/s. Its altitude (height) \( h \) after \( t \) seconds is given by \( h(t) = -16t^2 + 64t \). When does the projectile hit the ground?

a. 3 seconds  

b. 4 seconds  

c. 5 seconds  

d. 6 seconds

13. What are the dimensions of the largest rectangular field that can be enclosed with 100 m of wire?

a. 24 m \( \times \) 26 m  

b. 25 m \( \times \) 25 m  

c. 50 m \( \times \) 50 m  

d. 50 m \( \times \) 25 m

14. The batter hits the softball and it follows a path in which the height \( h \) is given by \( h(t) = -2t^2 + 8t + 3 \), where \( t \) is the time in seconds elapsed since the ball was pitched. What is the maximum height reached by the softball?

a. 11 m  

b. 12 m  

c. 13 m  

d. 14 m
Part II: Performance Task

Apply quadratic functions to solve the problem below. Show your solution.

Task 1 Being the first grandson, your grandparents decided to give you a rectangular field for your coming wedding. If you are given 200 m wires of fencing, what dimensions would you choose to get the maximum area?

a. List all the possible dimensions of the rectangular field.
b. Make a table of values for the possible dimensions.
c. Compute the area for each possible dimension.
d. What is the maximum area you obtained?
e. What are the dimensions of the maximum area you obtained?

Task 2 You are selling banana bread that costs Php 5 each. Each week, you have 50 customers. When you decrease the price by Php 1, you expect 30 customers to be added. What is the price of the banana bread that yields a maximum profit?

a. Analyze the problem.
b. What is the weekly sale if the cost of the banana bread is Php 5?
c. If the revenue \( R \) = number of bread \( x \) bread price. Write the equation of the quadratic function given the situation above.
d. What is the price that yields the maximum revenue?
e. Find the maximum revenue.

IV. LEARNING GOALS AND TARGETS

After going through this module, you should be able to demonstrate understanding of the key concepts of quadratic functions and be able to apply these to solve real-life problems. You will be able to formulate real-life problems involving quadratic functions, and solve them through a variety of techniques with accuracy.
Introduction to Quadratic Functions

What to KNOW

Let us start this lesson by recalling ways of representing a linear function. The knowledge and skills in doing this activity will help you a lot in understanding the quadratic function. In going over this lesson, you will be able to identify a quadratic function and represent it in different ways.

➤ Activity 1: Describe Me in Many Ways!

Perform this activity.

a. Observe the pattern and draw the 4th and 5th figures.

```
1  2  3   4   5
```

b. Use the table to illustrate the relation of the figure number to the number of blocks.

<table>
<thead>
<tr>
<th>Figure Number (x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blocks (y)</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td></td>
<td></td>
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</tbody>
</table>

c. Write the pattern observed from the table.

d. List the following:
   Set of ordered pairs _______________
   Domain _______________ Range _______________

e. What equation describes the pattern?

f. Graph the relation using the Cartesian Plane.

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</table>
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g. What are the independent and dependent variables?

h. What methods are used to describe the relation?
Activity 2: Parking Lot Problem

Solve the problem by following the procedure below.

Mr. Santos wants to enclose the rectangular parking lot beside his house by putting a wire fence on the three sides as shown in the figure. If the total length of the wire is 80 m, find the dimension of the parking lot that will enclose a maximum area.

Follow the procedure below:

a. In the figure above, if we let $w$ be the width and $l$ be the length, what is the expression for the sum of the measures of the three sides of the parking lot?

b. What is the length of the rectangle in terms of the width?

c. Express the area ($A$) of the parking lot in terms of the width.

d. Fill up the table by having some possible values of $w$ and the corresponding areas ($A$).

<table>
<thead>
<tr>
<th>Width ($w$)</th>
<th>Area ($A$)</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

e. What have you observed about the area ($A$) in relation to the width ($w$)?

f. What is the dependent variable? independent variable?

g. Compare the equation of a linear function with the equation you obtained.

h. From the table of values, plot the points and connect them using a smooth curve.

i. What do you observe about the graph?

j. Does the graph represent a linear function?
How did you find the preceding activity? I hope that you are now ready to learn about quadratic functions. These are functions that can be described by equations of the form 
\[ y = ax^2 + bx + c, \]
where, \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \). The highest power of the independent variable \( x \) is 2. Thus, the equation of a quadratic function is of degree 2.

➤ **Activity 3: Identify Me!**

State whether each of the following equations represents a quadratic function or not. Justify your answer.

<table>
<thead>
<tr>
<th>Equations</th>
<th>Yes or No</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( y = x^2 + 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. ( y = 2x - 10 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. ( y = 9 - 2x^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. ( y = 2^x + 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. ( y = 3x^3 + x^3 + 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. ( y = 2^x + 3x + 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. ( y = 2x^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. ( y = (x - 2)(x + 4) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. ( 0 = (x - 3)(x + 3) + x^2 - y )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. ( 3x^3 + y - 2x = 0 )</td>
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</tbody>
</table>

➤ **Activity 4: Compare Me!**

Follow the instructions below.

Consider the given functions \( f(x) = 2x + 1 \) and \( g(x) = x^2 + 2x - 1 \).

1. What kind of function is \( f(x) \)? \( g(x) \)?
2. Complete the following table of values using the indicated function.

\[
\begin{array}{c|c|c|c|c|c|c}
 x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
 \hline
 y & \multicolumn{7}{c|}{f(x) = 2x + 1} \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
 x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
 \hline
 y & \multicolumn{7}{c|}{g(x) = x^2 + 2x - 1} \\
\end{array}
\]

3. What are the differences between two adjacent \( x \)-values in each table?
4. Find the differences between each adjacent \( y \)-values in each table, and write them on the blanks provided.

\[
f(x) = 2x + 1 \\
g(x) = x^2 + 2x - 1
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

5. What do you observe?

6. How can you recognize a quadratic function when a table of values is given?

7. Using the table of values, graph the two functions and compare the results.

8. Compare the graph of linear function and quadratic function.

---

Did you enjoy the activity? You have seen that in a linear function, equal differences in \( x \) produce equal differences in \( y \). However, in a quadratic function, equal differences in \( x \) do not lead to equal first differences in \( y \); instead the second differences in \( y \) are equal. Notice also that the graph of a linear function is a straight line, while the graph of a quadratic function is a smooth curve. This smooth curve is a parabola.

In a quadratic function, equal differences in the independent variable \( x \) produce equal second differences in the dependent variable \( y \).
Illustrative example:
Let us consider \( y = x^2 - 4 \)

Differences in \( x \)

\[
\begin{array}{cccccccc}
  & 1 & 1 & 1 & 1 & 1 & 1 \\
  x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
  y &  5 &  0 & -3 & -4 & -3 &  0 &  5 \\
\end{array}
\]

First Differences in \( y \)

\[-5 \quad -3 \quad -1 \quad  1 \quad  3 \quad  5\]

Second Differences in \( y \)

\[2 \quad 2 \quad 2 \quad 2 \quad 2 \]

You have seen in the example above that in the quadratic function \( y = ax^2 + bx + c \), equal differences in \( x \) produce equal second differences in \( y \).

The previous activities familiarized you with the general form \( y = ax^2 + bx + c \) of a quadratic function. In your next activity, the standard form or vertex form \( y = a(x - h)^2 + k \) will be introduced. The standard form will be more convenient to use when working on problems involving the vertex of the graph of a quadratic function.

Study the illustrative examples presented below.

**Example 1:**

Express \( y = 3x^2 - 4x + 1 \) in the form \( y = a(x - h)^2 + k \) form and give the values of \( h \) and \( k \).

**Solution:**

\[ y = 3x^2 - 4x + 1 \]

\[ y = (3x^2 - 4x) + 1 \]

\[ y = 3 \left( x^2 - \frac{4}{3}x \right) + 1 \]

Group together the terms containing \( x \).

Factor out \( a \). Here, \( a = 3 \).

Complete the expression in parenthesis to make it a perfect square trinomial by adding the constant.

\[ 3 \left( \frac{4}{3} \right)^2 = 3 \left( \frac{2}{3} \right)^2 = 3 \left( \frac{4}{9} \right) = \frac{4}{3} \]

and subtracting the same value from the constant term.

\[ y = 3 \left( x^2 - \frac{4}{3}x + \left( \frac{2}{3} \right)^2 \right) + 1 - 3 \left( \frac{2}{3} \right)^2 \]

\[ = 3 \left( x - \frac{2}{3} \right)^2 + 1 - \frac{4}{3} \]

\[ = 3 \left( x - \frac{2}{3} \right)^2 + \frac{1}{3} \]
\[ y = \frac{3}{3} \left[ x^2 - \frac{4}{3} x + \frac{4}{9} \right] + 1 - \left( \frac{4}{3} \right) \] Simplify and express the perfect square trinomial as the square of a binomial.

\[ y = 3 \left( x - \frac{2}{3} \right)^2 - \frac{1}{3} \]

Hence, \( y = 3x^2 - 4x + 1 \) can be expressed as \( y = 3 \left( x - \frac{2}{3} \right)^2 - \frac{1}{3} \).

In this case, \( h = \frac{2}{3} \) and \( k = -\frac{1}{3} \).

**Example 2:**
Rewrite \( f(x) = ax^2 + bx + c \) in the form \( f(x) = a(x - h)^2 + k \).

**Solution:**
\[ y = (ax^2 + bx) + c \] Group together the terms containing \( x \).

\[ y = a \left( x^2 + \frac{b}{a} x \right) + c \] Factor out \( a \). Here, \( a = 1 \).

\[ y = a \left( x^2 + \frac{b}{a} x + \frac{b^2}{4a^2} \right) + c - \frac{b^2}{4a} \] Complete the expression in the parenthesis to make it a perfect square trinomial by adding a constant \( \frac{b^2}{4a} \) and subtracting the same value from the constant term.

\[ y = a \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a} \] Simplify and express the perfect square trinomial as the square of a binomial.

Hence, the vertex form is \( y = a \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a} \). Thus, \( h = \frac{-b}{2a} \) and \( k = \frac{4ac - b^2}{4a} \).

**Example 3:**
Rewrite \( f(x) = x^2 - 4x - 10 \) in the form \( f(x) = a(x - h)^2 + k \).

**Solution 1**
By completing the square:
\[ y = (x^2 - 4x) - 10 \] Group together the terms containing \( x \).

\[ y = (x^2 - 4x) - 10 \] Factor out \( a \). Here, \( a = 1 \).

\[ y = (x^2 - 4x + 4) - 10 - 4 \] Complete the expression in parenthesis to make it a perfect square trinomial by adding a constant \( \left( \frac{-4}{2} \right)^2 = 4 \) and subtracting the same value from the constant term.

\[ y = (x - 2)^2 - 14 \] Simplify and express the perfect square trinomial as the square of a binomial.
Solution 2

By applying the formula \( h = \frac{-b}{2a} \) and \( k = \frac{4ac - b^2}{4a} \):

In the equation \( y = x^2 - 4x - 10 \), \( a = 1 \), \( b = -4 \) and \( c = -10 \). Thus,

\[
\begin{align*}
    h &= \frac{-b}{2a} \\
    h &= \frac{-(-4)}{2(1)} \\
    h &= \frac{4}{2} \\
    h &= 2 \\

    k &= \frac{4ac - b^2}{4a} \\
    k &= \frac{4(1)(-10) - (-4)^2}{4(1)} \\
    k &= \frac{-40 - 16}{4} \\
    k &= -14
\end{align*}
\]

By substituting the solved values of \( h \) and \( k \) in \( y = a(x - h)^2 + k \), we obtain \( y = (x - 2)^2 - 14 \).

➤ Activity 5: Step by Step!

Work in pairs. Transform the given quadratic functions into the form \( y = a(x - h)^2 + k \) by following the steps below.
1. \( y = x^2 - 4x - 10 \)
2. \( y = 3x^2 - 4x + 1 \)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Group the terms containing ( x ).</td>
<td></td>
</tr>
<tr>
<td>2. Factor out ( a ).</td>
<td></td>
</tr>
<tr>
<td>3. Complete the expression in parenthesis to make it a perfect square trinomial.</td>
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<tr>
<td>4. Express the perfect square trinomial as the square of a binomial</td>
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<tr>
<td>5. Give the value of ( h )</td>
<td></td>
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<tr>
<td>6. Give the value of ( k )</td>
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</table>

Did you transform the quadratic function in the form \( y = a(x - h)^2 + k \)?
To transform a quadratic function from standard form \( y = a(x - h)^2 + k \) into general form, consider the examples below.

**Example 4:**
Rewrite the equation \( y = 3(x - 2)^2 + 4 \) in the general form \( y = ax^2 + bx + c \).

**Solution:**
- \( y = 3(x - 2)^2 + 4 \)
- Expand \((x - 2)^2\).
- \( y = 3(x^2 - 4x + 4) + 4 \)
- Multiply the perfect square trinomial by 3.
- \( y = 3x^2 - 12x + 12 + 4 \)
- Simplify and add 4.
- \( y = 3x^2 - 12x + 16 \)

**Example 5:**
Express \( f(x) = -2(3x - 1)^2 + 5x \) in the general form \( f(x) = ax^2 + bx + c \).

**Solution:**
- \( f(x) = -2(3x - 1)^2 + 5x \)
- \( f(x) = -2(9x^2 - 6x + 1) + 5x \)
- \( f(x) = -18x^2 + 12x - 2 + 5x \)
- \( f(x) = -18x^2 + 17x - 2 \)

➤ **Activity 6: Reversing the Process**
A. Rewrite \( y = 2(x - 1)^2 + 3 \) in the form \( y = ax^2 + bx + c \) by following the given steps.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Expand ((x - 1)^2)</td>
<td></td>
</tr>
<tr>
<td>2. Multiply the perfect square trinomial by 2</td>
<td></td>
</tr>
<tr>
<td>3. Simplify</td>
<td></td>
</tr>
<tr>
<td>4. Add 3</td>
<td></td>
</tr>
<tr>
<td>5. Result</td>
<td></td>
</tr>
</tbody>
</table>

B. Apply the above steps in transforming the following quadratic functions into the general form.
1. \( y = 2(x - 4)^2 + 5 \)
2. \( y = 3\left(x - \frac{1}{2}\right)x + 1 \)

**Did you transform the quadratic function into the form \( y = ax^2 + bx + c \)?**

132
Your goal in this section is to master the skills in identifying the quadratic function and transforming it into different forms. Towards the end of this module, you will be encouraged to apply these skills in solving real-life problems.

➤ Activity 7: Where Do You Belong?

Put the letter of the given equation in the diagram below where you think it belongs.

- a. \( y = x^2 - 1 \)
- b. \( y = x \)
- c. \( 2x^2 - 2x + 1 = y \)
- d. \( 3x - 1 + y = 0 \)
- e. \( y = (2x + 3)(x - 1) \)
- f. \( y = x^3 + 1 \)
- g. \( 2^2 + x = y \)
- h. \( y = 3^x + 2x \)
- i. \( 3x + x^2 = y \)
- j. \( 2x(x - 3) - y = 0 \)

How did you classify each of the given functions?
What similarities do you see in quadratic functions? in linear functions?
How do a quadratic function and a linear function differ?
What makes a function quadratic?

➤ Activity 8: Quadratic or Not

Study the patterns below. Indicate whether the pattern described by the figures is quadratic or not.

- a. 

Determine the relationship between the number of blocks in the bottom row and the total number of blocks.
What relationship exists between the two numbers?
b. Determine the relationship between the number of blocks in the bottom row and the total number of blocks. What relationship exists between the two numbers?

c. Determine the relationship between the number of blocks in the bottom row and the total number of blocks in the figure. What relationship exists between the two numbers?

➤ Activity 9: It’s Your Turn

Match the given quadratic function $y = ax^2 + bx + c$ to its equivalent standard form $y = a(x - h)^2 + k$. 

- $y = x^2 - x + \frac{13}{4}$
- $y = \frac{1}{2}x^2 - 3x + 3$
- $y = -2x^2 + 12x - 17$
- $y = (x - 2)^2 - 3$
- $y = 2(x - 1)^2 + 2$
- $y = -2(x - 3)^2 + 1$
What mathematical concepts did you use in doing the transformation?
Explain how the quadratic function in the form $y = ax^2 + bx + c$ can be transformed into the form $y = a(x - h)^2 + k$.

➤ Activity 10: The Hidden Message

Write the indicated letter of the quadratic function in the form $y = a(x - h)^2 + k$ into the box that corresponds to its equivalent general form $y = ax^2 + bx + c$.

I $y = (x - 1)^2 - 4$
T $y = (x - 1)^2 - 16$

S $y = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$
F $y = (x - 3)^2 + 5$

E $y = \left(x - \frac{2}{3}\right)^2 + 2$
M $y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$

A $y = 3(x + 2)^2 - \frac{1}{2}$
U $y = -2(x - 3)^2 + 1$

N $y = (x - 0)^2 - 36$
H $y = 2(x + 1)^2 - 2$

DIALOG BOX:

$y = x^2 - x + \frac{7}{4}$

$y = 3x^2 + 12x + \frac{23}{2}$

$y = x^2 - 2x - 15$

$y = 2x^2 + 4x$

$y = x^2 - 2x - 3$
How is the square of a binomial obtained without using the long method of multiplication? Explain how the quadratic function in the form \( y = a(x - h)^2 + k \) can be transformed into the form \( y = ax^2 + bx + c \).

➤ Activity 11: Hit or Miss!

Work in pairs. Solve this problem and show your solution.

An antenna is 5 m high and 150 m from the firing place. Suppose the path of the bullet shot from the firing place is determined by the equation \( y = -\frac{1}{1500}x^2 + \frac{2}{15}x \), where \( x \) is the distance (in meters) of the bullet from the firing place and \( y \) is its height. Will the bullet go over the antenna? If yes/no, show your justification.

What to REFLECT and UNDERSTAND

Your goal in this section is to have a better understanding of the mathematical concepts about quadratic functions. The activities provided for you in this section aim to apply the different concepts that you have learned from the previous activities.
➤ **Activity 12: Inside Outside Circle (Kagan, 1994)**

1. Form a group of 20 members and arrange yourselves by following the formation below.
2. Listen to your teacher regarding the procedures of the activity.

Guide Questions/Topics for the activity.
1. What is a quadratic function?
2. How do you differentiate the equation of a quadratic function from that of a linear function?
3. Describe the graph of a linear function and the graph of a quadratic function.
4. Given a table of values, how can you determine if the table represents a quadratic function?

➤ **Activity 13: Combination Notes**

A. In the oval callout, describe the ways of recognizing a quadratic function.
B. In the oval callout, make an illustrative example of the indicated mathematical concept.

<table>
<thead>
<tr>
<th>Concepts</th>
<th>Illustrative Example</th>
</tr>
</thead>
</table>
| Transforming a quadratic function in the form \( y = ax^2 + bx + c \) into the form \( y = a(x - h)^2 + k \). | ![Illustrative Example](image)

Based on what you have learned in the preceding activity, you are now ready to apply the concepts that you have learned in other contexts.

➤ Activity 14: **Find My Pattern!**

Group yourselves into 5. Perform the activity below.

Consider the set of figures below. Study the relationship between the term number and the number of unit triangles formed. What is the pattern? Describe the patterns through a table of values, graph, and equation. How many triangles are there in the 25th term?

<table>
<thead>
<tr>
<th>Term number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Figure 1" /></td>
<td><img src="image" alt="Figure 2" /></td>
<td><img src="image" alt="Figure 3" /></td>
<td></td>
</tr>
</tbody>
</table>

What to **TRANSFER**

The goal of this section is for you to apply what you have learned in a real-life situation. You will be given a task which will demonstrate your understanding of the lesson.
➤ Activity 15: Investigate!

**Problem.** You are given 50 m of fencing materials. Your task is to make a rectangular garden whose area is a maximum. Find the dimensions of such a rectangle. Explain your solution.

➤ Activity 16: Explore More!

Give at least three parabolic designs that you see in your community. Then create your own design.

**Summary/Synthesis/Generalization**

This lesson introduced quadratic functions. The lesson provided you with opportunities to describe a quadratic function in terms of its equation, graph, and table of values. You were given a chance to compare and see the difference between quadratic functions and linear or other functions.
Graphs of Quadratic Functions

What to KNOW

Let’s start this lesson by generating a table of values of quadratic functions and plotting the points on the coordinate plane. You will investigate the properties of the graph through guided questions. As you go through this lesson, keep on thinking about this question: How can the graph of a quadratic function be used to solve real-life problems?

➤ Activity 1: Describe My Paths!

Follow the procedure in doing the activity.

a. Given the quadratic functions \(y = x^2 - 2x - 3\) and \(y = -x^2 + 4x - 1\), transform them into the form \(y = a(x - h)^2 + k\).

\[
\begin{align*}
y &= x^2 - 2x - 3 \\
y &= -x^2 + 4x - 1
\end{align*}
\]

b. Complete the table of values for \(x\) and \(y\).

\[
\begin{array}{c|cccccccc}
\hline
x & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
y & & & & & & & & & \\
\hline
\end{array}
\]

\[
\begin{array}{c|cccccccc}
\hline
x & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
y & & & & & & & & & \\
\hline
\end{array}
\]

c. Sketch the graph on the Cartesian plane.
d. What have you observed about the opening of the curves? Do you have any idea where you can relate the opening of the curves?

e. Which of the 2 quadratic functions has a minimum point? maximum point? Indicate below.

<table>
<thead>
<tr>
<th>Quadratic Function</th>
<th>Vertex (Turning Point)</th>
<th>Maximum or Minimum Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2 - 2x - 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = -x^2 + 4x - 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f. Observe each graph. Can you draw a line that divides the graph in such a way that one part is a reflection of the other part? If there is any, determine the equation of the line?

g. Take a closer look at the minimum point or the maximum point and try to relate it to the values of $h$ and $k$ in the equation $y = a(x - h)^2 + k$ of the function. Write your observations.

h. Can you identify the domain and range of the functions?

- $y = x^2 - 2x - 3$  Domain: __________  Range: __________
- $y = -x^2 + 4x - 1$  Domain: __________  Range: __________

Did you enjoy the activity? To better understand the properties of the graph of a quadratic function, study some key concepts below.

The graph of a quadratic function $y = ax^2 + bx + c$ is called **parabola**. You have noticed that the parabola opens **upward** or **downward**. It has a turning point called **vertex** which is either the lowest point or the highest point of the graph. If the value of $a > 0$, the parabola opens upward and has a **minimum point**. If $a < 0$, the parabola opens downward and has a **maximum point**. There is a line called the **axis of symmetry** which divides the graph into two parts such that one-half of the graph is a reflection of the other half. If the quadratic function is expressed in the form $y = a(x - h)^2 + k$, the vertex is the point $(h, k)$. The line $x = h$ is the axis of symmetry and $k$ is the minimum or maximum value of the function.

The **domain** of a quadratic function is the set of all real numbers. The **range** depends on whether the parabola opens upward or downward. If it opens upward, the range is the set $\{y : y \geq k\}$; if it opens downward, then the range is the set $\{y : y \leq k\}$. 
**Activity 2: Draw Me!**

Draw the graph of the quadratic function $y = x^2 - 4x + 1$ by following the steps below.

1. Find the vertex and the line of symmetry by expressing the function in the form $y = a(x - h)^2 + k$ or by using the formula $h = \frac{-b}{2a}$; $k = \frac{4ac - b^2}{4a}$ if the given quadratic function is in general form.

2. On one side of the line of symmetry, choose at least one value of $x$ and compute the value of $y$.
   Coordinates of points: _____________________________________________

3. Similarly, choose at least one value of $x$ on the other side and compute the value of $y$.
   Coordinates of points: _____________________________________________

4. Plot the points and connect them by a smooth curve.

**Activity 3: Play and Learn!**

Work in a group of 5 members. Solve the puzzle and do the activity.

*Problem:* Think of a number less than 20. Subtract this number from 20 and multiply the difference by twice the original number. What is the number that will give the largest product?

The first group who gives the largest product wins the game.

a. Record your answer on the table below:

<table>
<thead>
<tr>
<th>Number $(n)$</th>
<th>Product $(P)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Draw the graph.

c. Find the vertex and compare it to your answer in the puzzle.

d. If $n$ is the number you are thinking, then how can you express the other number, which is the difference of 20 and the number you are thinking of?

e. What is the product $(P)$ of the two numbers? Formulate the equation.

f. What kind of function is represented by the equation?

g. Express it in standard form.

h. What is the largest product?

i. What is the number that will give the largest product?

j. Study the graph and try to relate the answer you obtained in the puzzle to the vertex of the graph. Write your observation.
Activity 4: To the Left, to the Right!  
Put Me Up, Put Me Down!

Form groups of 5 members each and perform this activity.

A. Draw the graphs of the following quadratic functions on the same coordinate plane.
1. \( y = x^2 \)
2. \( y = 2x^2 \)
3. \( y = 3x^2 \)
4. \( y = \frac{1}{2}x^2 \)
5. \( y = \frac{1}{3}x^2 \)
6. \( y = -x^2 \)
7. \( y = -2x^2 \)

   a. Analyze the graphs.
   b. What do you notice about the shape of the graph of the quadratic function \( y = ax^2 \)?
   c. What happens to the graph as the value of \( a \) becomes larger?
   d. What happens when \( 0 < a < 1 \)?
   e. What happens when \( a < 0 \) ? \( a > 0 \) ?
   f. Summarize your observations.

B. Draw the graphs of the following functions.
1. \( y = x^2 \)
2. \( y = (x - 2)^2 \)
3. \( y = (x + 2)^2 \)
4. \( y = (x + 1)^2 \)
5. \( y = (x - 1)^2 \)

   a. Analyze the graphs.
   b. What do you notice about the graphs of quadratic functions whose equations are of the form \( y = (x - h)^2 \)?
   c. How would you compare the graph of \( y = (x - h)^2 \) and that of \( y = x^2 \)?
   d. Discuss your ideas and observations.
C. Draw the graphs of the following quadratic functions:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( y = x^2 )</td>
</tr>
<tr>
<td>2.</td>
<td>( y = x^2 + 2 )</td>
</tr>
<tr>
<td>3.</td>
<td>( y = x^2 - 2 )</td>
</tr>
<tr>
<td>4.</td>
<td>( y = x^2 - 3 )</td>
</tr>
<tr>
<td>5.</td>
<td>( y = x^2 + 3 )</td>
</tr>
</tbody>
</table>

a. Analyze the graphs.
b. What do you notice about the graphs of quadratic functions whose equations are of the form \( y = x^2 + k \)?
c. How would you compare the graph of \( y = x^2 + k \) and that of \( y = x^2 \) when the vertex is above the origin? below the origin?
d. What conclusion can you give based on your observations?

D. Draw the graphs of the following quadratic functions:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( y = (x - 2)^2 + 4 )</td>
</tr>
<tr>
<td>2.</td>
<td>( y = (x + 3)^2 - 4 )</td>
</tr>
<tr>
<td>3.</td>
<td>( y = (x - 1)^2 - 3 )</td>
</tr>
<tr>
<td>4.</td>
<td>( y = (x + 4)^2 + 5 )</td>
</tr>
<tr>
<td>5.</td>
<td>( y = (x + 2)^2 - 2 )</td>
</tr>
</tbody>
</table>

a. Analyze the graphs.
b. What is the effect of the variables \( h \) and \( k \) on the graph of \( y = (x - h)^2 + k \) as compared to the graph of \( y = x^2 \)?
c. Make your generalization on the graph of \( y = (x - h)^2 + k \).

---

Did you enjoy the activity? To better understand the transformation of the graph of a quadratic function, read some key concepts.
In the graph of \( y = ax^2 + bx + c \), the larger the \( |a| \) is, the narrower is the graph.

For \( a > 0 \), the parabola opens upward.
To graph \( y = a(x - h)^2 \), slide the graph of \( y = ax^2 \) horizontally \( h \) units. If \( h > 0 \), slide it to the right, if \( h < 0 \), slide it to the left. The graph has vertex \((h, 0)\) and its axis is the line \( x = h \).

To graph \( y = ax^2 + k \), slide the graph of \( y = ax^2 \) vertically \( k \) units. If \( k > 0 \) slide it upward; if \( k < 0 \), slide it downward. The graph has vertex \((0, k)\) and its axis of symmetry is the line \( x = 0 \) (\( y \)-axis).

To graph \( y = a(x - h)^2 + k \), slide the graph of \( y = ax^2 \) horizontally \( h \) units and vertically \( k \) units. The graph has a vertex \((h, k)\) and its axis of symmetry is the line \( x = h \).
If \( a < 0 \), the parabola opens downward. The same procedure can be applied in transforming the graph of a quadratic function.

**Vertex of the graph of a quadratic function:**
In standard form \( f(x) = a(x - h)^2 + k \), the vertex \((h, k)\) can be directly obtained from the values of \( h \) and \( k \).

In general form \( f(x) = ax^2 + bx + c \), the vertex \((h, k)\) can be obtained using the formulas
\[
h = \frac{-b}{2a} \quad \text{and} \quad k = \frac{4ac - b^2}{4a}.
\]

**What to PROCESS**

Your goal in this section is to apply the mathematical concepts that you have learned in graphing quadratic functions. Use these mathematical concepts to perform the provided activities in this section.
Activity 5: Draw and Describe Me!

Sketch the graph of each quadratic function and identify the vertex, domain, range, and the opening of the graph. State whether the vertex is a minimum or a maximum point, and write the equation of its axis of symmetry.

1. \( f(x) = x^2 \)

   Vertex _____________
   Opening of the graph _____________
   Vertex is a _____________ point
   Equation of the axis of symmetry ______
   Domain: _____ Range: _____

2. \( f(x) = 2x^2 + 4x - 3 \)

   Vertex _____________
   Opening of the graph _____________
   Vertex is a _____________ point
   Equation of the axis of symmetry ______
   Domain: _____ Range: _____

3. \( f(x) = \frac{1}{2}x^2 + 2 \)

   Vertex _____________
   Opening of the graph _____________
   Vertex is a _____________ point
   Equation of the axis of symmetry ______
   Domain: _____ Range: _____
4. \( f(x) = -x^2 - 2x - 3 \)

Vertex _____________
Opening of the graph _____________
Vertex is a _____________ point
Equation of the axis of symmetry ______
Domain: _____ Range: _____

5. \( f(x) = (x + 2)^2 + 3 \)

Vertex _____________
Opening of the graph _____________
Vertex is a _____________ point
Equation of the axis of symmetry ______
Domain: _____ Range: _____

6. \( f(x) = 2(x - 2)^2 \)

Vertex _____________
Opening of the graph _____________
Vertex is a _____________ point
Equation of the axis of symmetry ______
Domain: _____ Range: _____

7. \( f(x) = -2x^2 - 2 \)

Vertex _____________
Opening of the graph _____________
Vertex is a _____________ point
Equation of the axis of symmetry ______
Domain: _____ Range: _____
How did you find the activity? Explain the procedure on how to draw the graph of a quadratic function.

➤ Activity 6: Hit the Volleyball

Carl Allan hit the volleyball at 3 ft above the ground with an initial velocity of 32 ft/sec. The path of the ball is given by the function $S(t) = -16t^2 + 32t + 3$, where $S$ is the height of the ball at $t$ seconds. What is the maximum height reached by the ball?

a. What kind of function is used to model the path of the volleyball?

b. Draw the path of the volleyball and observe the curve.

c. What is the maximum height reached by the ball?

d. What is represented by the maximum point of the graph?

➤ Activity 7: Match or Mismatch!

Decide whether the given graph is a match or a mismatch with the indicated equation of quadratic function. Write match if the graph corresponds with the correct equation. Otherwise, indicate the correct equation of the quadratic function.

1. $y = (x + 4)^2$
Share the technique you used to determine whether the graph and the equation of the quadratic function are matched or mismatched.

What characteristics of a quadratic function did you apply in doing the activity?
Activity 8: Translate Me!

The graph of \( f(x) = 2x^2 \) is shown below. Based on this graph, sketch the graphs of the following quadratic functions in the same coordinate system.

a. \( f(x) = 2x^2 + 3 \)

b. \( f(x) = 2(x - 3)^2 - 1 \)

c. \( f(x) = 2(x + 2)^2 \)

d. \( f(x) = -2x^2 + 3 \)

e. \( f(x) = 2(x + 1)^2 - 2 \)

f. \( f(x) = 2(x + 4)^2 \)

g. \( f(x) = -2x^2 - 1 \)

h. \( f(x) = 2(x + 3)^2 + 5 \)

i. \( f(x) = 2(x - 3)^2 - 2 \)

j. \( f(x) = -2(x - 1)^2 \)

How did you find the activity?

Describe the movement of the graph for each quadratic function.

What to REFLECT and UNDERSTAND

Your goal in this section is to have a deeper understanding of the graph of quadratic functions. The activities provided for you in this section will be of great help to enable you to apply the concepts in different contexts.
Activity 9: Let’s Analyze!

Analyze the problem and answer the given questions.

Problem 1: A ball on the playing ground was kicked by Carl Jasper. The parabolic path of the ball is traced by the graph below. Distance is given in meters.

Questions:

a. How would you describe the graph?
b. What is the initial height of the ball?
c. What is the maximum height reached by the ball?
d. Determine the horizontal distance that corresponds to the maximum distance.
e. Determine/Approximate the height of the ball after it has travelled 2 meters horizontally.
f. How far does the ball travel horizontally before it hits the ground?

Problem 2: The path when a stone is thrown can be modelled by \( y = -16x^2 + 10x + 4 \), where \( y \) (in feet) is the height of the stone \( x \) seconds after it is released.

a. Graph the function.
b. Determine the maximum height reached by the stone.
c. How long will it take the stone to reach its maximum height?
Activity 10: Clock Partner Activity

Write your name on your clock. Make an appointment with 12 of your classmates, one for each hour on the clock. Be sure you both record the appointment on your clock. Make an appointment only if there is an open slot at that hour on both your clock.

Use your clock partner to discuss the following questions.

<table>
<thead>
<tr>
<th>Time</th>
<th>Topics/Questions to be discussed/answered</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:00</td>
<td>How can you describe the graph of a quadratic function?</td>
</tr>
<tr>
<td>5:00</td>
<td>Tell something about the axis of symmetry of a parabola.</td>
</tr>
<tr>
<td>12:00</td>
<td>When do we say that the graph has a minimum/maximum value or point?</td>
</tr>
<tr>
<td>3:00</td>
<td>What does the vertex imply?</td>
</tr>
<tr>
<td>5:00</td>
<td>How can you determine the opening of the parabola?</td>
</tr>
<tr>
<td>6:00</td>
<td>How would you compare the graph of $y = a(x – h)^2$ and that of $y = ax^2$?</td>
</tr>
<tr>
<td>11:00</td>
<td>How would you compare the graph of $y = x^2 + k$ with that of $y = x^2$ when the vertex is above the origin? below the origin?</td>
</tr>
<tr>
<td>1:00</td>
<td>How do the values of $h$ and $k$ in $y = a(x - h)^2 + k$ affect the graph of $y = ax^2$?</td>
</tr>
</tbody>
</table>

Activity 11: Combination Notes

Tell something about what you have learned.

- What are the properties of the graph of a quadratic function? Discuss briefly.
- Enumerate the steps in graphing a quadratic function.
- How can you determine the vertex of a quadratic function?
Activity 12: Which of Which?

Answer each of the following questions.

1. Which has the larger area?
   a. A rectangle whose dimensions are 25 m by 20 m
   b. The largest possible area of a rectangle to be enclosed if the perimeter is 50 m

2. Which has the lower vertex?
   a. \( y = x^2 + 2x + 3 \)
   b. \( y = x^2 - 4x + 7 \)

3. Which has the higher vertex?
   a. \( y = -x^2 - 6x + 15 \)
   b. \( y = -x^2 + 6 \)
Activity 13: ABC in Math!

Play and learn with this activity.

In activity 3, you have learned the effects of variables $a$, $h$, and $k$ in the graph of $y = a(x - h)^2 + k$ as compared to the graph of $y = ax^2$. Now, try to investigate the effect of variables $a$, $b$, and $c$ in the graph of the quadratic function $y = ax^2 + bx + c$.

Did you enjoy the activities? I hope that you learned a lot in this section and you are now ready to apply the mathematical concepts you gained from all the activities and discussions.

What to TRANSFER

In this section, you will be given a task wherein you will apply what you have learned in the previous sections. Your performance and output will show evidence of learning.

Activity 14: Quadratic Design

Goal: Your task is to design a curtain in a small restaurant that involves a quadratic curve.

Role: Interior Designer

Audience: Restaurant Owner

Situation: Mr. Andal, the owner of a restaurant wants to impress some of the visitors, as target clients, in the coming wedding of his friend. As a venue of the reception, Mr. Andal wants a new ambience in his restaurant. Mr. Andal requested you, as interior designer, to help him to change the interior of the restaurant particularly the design of the curtains. Mr. Andal wants you to use parabolic curves in your design. Map out the appearance of the proposed design for the curtains in his 20 by 7 meters restaurant and estimate the approximate budget requirements for the cost of materials based on the height of the curve design.

Product: Proposed plan for the curtain including the proposed budget based on the height of the curve design.
Standards for Assessment:
You will be graded based on the rubric designed suitable for your task and performance.

Activity 15: Webquest Activity. Math is all around.

Make a simple presentation of world famous parabolic arches.

Task:
1. Begin the activity by forming a group of 5 members. Choose someone you can depend on to work diligently and to do his fair share of work.
2. In your free time, start surfing the net for world famous parabolic arches. As you search, keep a record of where you go, and what you find on the site.
3. Complete the project by organizing the data you collected, including the name of the architect and the purpose of creating the design.
4. Once you have completed the data, present it to the class in a creative manner. You can use any of the following but not limited to them.
   - Multimedia presentation
   - Webpages
   - Posters
5. You will be assessed based on the rubric for this activity.

Summary/Synthesis/Generalization

This lesson was about graphs of quadratic functions. The lesson was able to equip you with ample knowledge on the properties of the graph of quadratic functions. You were made to experience graphing quadratic functions and their transformations. You were given opportunities to solve real-life problems using graphs of quadratic functions and to create designs out of them.
Finding the Equation of a Quadratic Function

What to KNOW

Let’s begin this lesson by recalling the methods of finding the roots of quadratic equations. Then, relate them with the zeros of quadratic functions. In this lesson, you will be able to formulate patterns and relationship regarding quadratic functions. Furthermore, you will be able to solve real-life problems involving equations of quadratic functions.

➤ Activity 1: Give Me My Roots!

Given a quadratic equation \( x^2 – x – 6 = 0 \), find the roots in three methods.

- Factoring

- Quadratic Formula

- Completing the Square

Did you find the roots in 3 different ways? Your skills in finding the roots will also be the methods you will be using in finding the zeros of quadratic functions. To better understand the zeros of quadratic functions and the procedure in finding them, study the mathematical concepts below.

A value of \( x \) that satisfies the quadratic equation \( ax^2 + bx + c = 0 \) is called a root of the equation.
Activity 2: What Are My Zeros?

Perform this activity and answer the guided questions.

Examine the graph of the quadratic function $y = x^2 - 2x - 3$

a. How would you describe the graph?
b. Give the vertex of the parabola and its axis of symmetry.
c. At what values of $x$ does the graph intersect the $x$-axis?
d. What do you call these $x$-coordinates where the curve crosses the $x$-axis?
e. What is the value of $y$ at these values of $x$?

How did you find the activity? To better understand the zeros of a function, study some key concepts below.

The graph of a quadratic function is a parabola. A parabola can cross the $x$-axis once, twice, or never. The $x$-coordinates of these points of intersection are called x-intercepts. Let us consider the graph of the quadratic function $y = x^2 - x - 6$. It shows that the curve crosses the $x$-axis at 3 and -2. These are the $x$-intercepts of the graph of the function. Similarly, 3 and -2 are the zeros of the function since these are the values of $x$ when $y$ equals 0. These zeros of the function can be determined by setting $y$ to 0 and solving the resulting equation through different algebraic methods.
Example 1

Find the zeros of the quadratic function \( y = x^2 - 3x + 2 \) by factoring method.

Solution:
Set \( y = 0 \). Thus,
\[
0 = x^2 - 3x + 2
\]
\[
0 = (x - 2)(x - 1)
\]
\[
x - 2 = 0 \quad \text{or} \quad x - 1 = 0
\]
Then \( x = 2 \) and \( x = 1 \)
The zeros of \( y = x^2 - 3x + 2 \) are 2 and 1.

Example 2

Find the zeros of the quadratic function \( y = x^2 + 4x - 2 \) using the completing the square method.

Solution:
Set \( y = 0 \). Thus,
\[
x^2 + 4x - 2 = 0
\]
\[
x^2 + 4x = 2
\]
\[
x^2 + 4x + 4 = 2 + 4
\]
\[
(x + 2)^2 = 6
\]
\[
x + 2 = \pm \sqrt{6}
\]
\[
x = -2 \pm \sqrt{6}
\]
The zeros of \( y = x^2 + 4x - 2 \) are \(-2 + \sqrt{6}\) and \(-2 - \sqrt{6}\). 
Example 3.

Find the zeros of the quadratic function \( f(x) = x^2 + x - 12 \) using the quadratic formula.

**Solution:**

Set \( y = 0 \).

In \( y = x^2 + x - 12 \), \( a = 1 \), \( b = 1 \), and \( c = -12 \).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Use the quadratic formula.

\[
x = \frac{-1 \pm \sqrt{1 + 48}}{2}
\]

Substitute the values of \( a \), \( b \), and \( c \).

\[
x = \frac{-1 \pm \sqrt{49}}{2}
\]

Simplify.

\[
x = \frac{-1 \pm 7}{2}
\]

\[
x = \frac{6}{2} \quad \text{or} \quad x = \frac{-8}{2}
\]

\[
x = 3 \quad \text{or} \quad x = -4
\]

The zeros of \( f(x) = x^2 + x - 12 \) are 3 and -4.

➤ **Activity 3: What’s My Rule!**

Work in groups of three (3) members each. Perform this activity.

The table below corresponds to a quadratic function. Examine it.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-1</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-29</td>
<td>-5</td>
<td>3</td>
<td>1</td>
<td>-5</td>
</tr>
</tbody>
</table>

➤ **Activity 3.A**

a. Plot the points and study the graph. What have you observed?

b. What are the zeros of the quadratic function? How can you identify them?

c. If the zeros are \( r_1 \) and \( r_2 \), express the equation of the quadratic function using

\[
f(x) = a(x - r_1)(x - r_2), \text{ where } a \text{ is any non-zero constant.}
\]

d. What is the quadratic equation that corresponds to the table?
Can you think of another way to determine the equation of the quadratic function from the table of values?

What if the table of values does not have the zero/s of the quadratic function? How can you derive its equation?

➤ Activity 3.B

The table of values below describes a quadratic function. Find the equation of the quadratic function by following the given procedure.

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-1</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-29</td>
<td>-5</td>
<td>3</td>
<td>1</td>
<td>-5</td>
</tr>
</tbody>
</table>

a. Substitute three (3) ordered pairs \((x, y)\) in \(y = ax^2 + bx + c\)
b. What are the three equations you came up with?

___________________, __________________, ___________________
c. Solve for the values of \(a\), \(b\), and \(c\).
d. Write the equation of the quadratic function \(y = ax^2 + bx + c\).

How did you obtain the three equations?
What do you call the three equations?
How did you solve for the values of \(a\), \(b\), and \(c\) from the three equations?
How can you obtain the equation of a quadratic function from a table of values?

Did you get the equation of the quadratic function correctly in the activity? You can go over the illustrative examples below to better understand the procedure on how to determine the equation of a quadratic function given the table of values.

Study the illustrative examples below.

**Illustrative example 1**

Find a quadratic function whose zeros are -1 and 4.

**Solution:** If the zeros are -1 and 4, then

\[ x = -1 \text{ or } x = 4 \]

It follows that

\[ x + 1 = 0 \text{ or } x - 4 = 0, \text{ then} \]

\[ (x + 1) (x - 4) = 0 \]

\[ x^2 - 3x - 4 = 0 \]
The equation of the quadratic function \( f(x) = (x^2 - 3x - 4) \) is not unique since there are other quadratic functions whose zeros are -1 and 4 like \( f(x) = 2x^2 - 6x - 8 \), \( f(x) = 3x^2 - 9x - 12 \) and many more. These equations of quadratic functions are obtained by multiplying the right-hand side of the equation by a nonzero constant. Thus, the answer is \( f(x) = a(x^2 - 3x - 4) \) where \( a \) is any nonzero constant.

**Illustrative example 2**

Determine the equation of the quadratic function represented by the table of values below.

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>24</td>
<td>16</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

**Solution:**

Notice that you can’t find any zeros from the given table of values. In this case, take any three ordered pairs from the table, and use these as the values of \( x \) and \( y \) in the equation \( y = ax^2 + bx + c \). Let’s say

using point (1, 4) \( \quad 4 = a(1)^2 + b(1) + c \)

\( 4 = a + b + c \) \( \rightarrow \) equation 1

using point (-1, 10) \( \quad 10 = a(-1)^2 + b(-1) + c \)

\( 10 = a - b + c \) \( \rightarrow \) equation 2

using point (2, 4) \( \quad 4 = a(2)^2 + b(2) + c \)

\( 4 = 4a + 2b + c \) \( \rightarrow \) equation 3

We obtain a system of 3 equations in \( a, b, \) and \( c \).

Add corresponding terms in eq.1 and eq. 2 to eliminate \( b \)

\[ \begin{align*}
\text{eq. 1} + \text{eq. 2} & \quad 4 = a + b + c \\
10 &= a - b + c \\
14 &= 2a + 2c \quad \rightarrow \text{ equation 4}
\end{align*} \]

Multiply the terms in eq. 2 by 2 and add corresponding terms in eq. 3 to eliminate \( b \)

\[ \begin{align*}
2(\text{eq. 2}) + \text{eq. 3} & \quad 20 = 2a - 2b + 2c \\
4 &= 4a + 2b + c \\
24 &= 6a + 3c \quad \rightarrow \text{ equation 5}
\end{align*} \]

Notice that equation 4 and equation 5 constitute a system of linear equations in two variables. To solve for \( c \), multiply the terms in equation 4 by 3 and subtract corresponding terms in equation 5.

\[ \begin{align*}
3(\text{eq. 4}) - \text{eq. 5} & \quad 42 = 6a + 6c \\
24 &= 6a + 3c \\
18 &= 3c \\
\hline
c &= 6
\end{align*} \]
Substitute the value of $c$ in equation 4 and solve for $a$.

$14 = 2a + 2(6)$
$14 = 2a + 12$
$2a = 14 – 12$
$a = 1$

Substitute the value of $c$ and $a$ in equation 1 and solve for $b$.

$4 = a + b + c$
$4 = 1 + b + 6$
$4 = 7 + b$
$b = 4 – 7$
$b = -3$

Thus, $a = 1$, $b = -3$, and $c = 6$. Substitute these in $f(x) = ax^2 + bx + c$; the quadratic function is $f(x) = x^2 – 3x + 6$.

➤ Activity 4: Pattern from Curve!

Work in pairs. Determine the equation of the quadratic function given the graph by following the steps below.

Study the graph of the quadratic function below.

1. What is the opening of the parabola? What does it imply regarding the value of $a$?
2. Identify the coordinates of the vertex.
3. Identify coordinates of any point on the parabola.
4. In the form of quadratic function $y = a(x – h)^2 + k$, substitute the coordinates of the point you have taken in the variables $x$ and $y$ and the $x$-coordinates and $y$-coordinate of the vertex in place of $h$ and $k$, respectively.
5. Solve for the value of $a$.
6. Get the equation of a quadratic in the form $y = a(x – h)^2 + k$ by substituting the obtained value of $a$ and the $x$ and $y$ coordinates of the vertex in $h$ and $k$ respectively.
How did you find the activity? Study the mathematical concepts below to have a clear understanding of deriving the quadratic equation from the graph.

When the vertex and any point on the parabola are clearly seen, the equation of the quadratic function can easily be determined by using the form of a quadratic function $y = a(x - h)^2 + k$.

**Illustrative example 1**

Find the equation of the quadratic function determined from the graph below.

![Graph of a quadratic function with vertex (2, -3) and point (5, 0)](

**Solution:**

The vertex of the graph of the quadratic function is (2, -3). The graph passes through the point (5, 0). By replacing $x$ and $y$ with 5 and 0, respectively, and $h$ and $k$ with 2 and -3, respectively, we have

\[
y = a(x - h)^2 + k \\
0 = a(5 - 2)^2 + (-3) \\
0 = a(3)^2 - 3 \\
3 = 9a \\
a = \frac{1}{3}
\]

Thus, the quadratic equation is $y = \frac{1}{3}(x - 2)^2 - 3$ or $y = \frac{1}{3}x^2 - \frac{4}{3}x - \frac{5}{3}$.

Aside from the method presented above, you can also determine the equation of a quadratic function by getting the coordinates of any 3 points lying on the graph. You can follow the steps in finding the equation of a quadratic function using this method by following the illustrative example presented previously in this section.
Activity 5: Give My Equation!

Perform the activity.

A. Study the example below in finding the zeros of the quadratic function and try to reverse the process to find the solution of the problem indicated in the table on the right.

Find the zeros of \( f(x) = 6x^2 - 7x - 3 \) using factoring.

Solution:

\[
\begin{align*}
0 &= 6x^2 - 7x - 3 \\
0 &= (2x - 3)(3x + 1) \\
2x - 3 &= 0 \text{ or } 3x + 1 = 0 \\
Then \ x &= \frac{3}{2} \text{ and } x = -\frac{1}{3}. \\
The zeros are \( \frac{3}{2} \) and \( -\frac{1}{3} \).
\end{align*}
\]

If the zeros of the quadratic function are 1 and 2, find the equation.

Note: \( f(x) = a(x - r_1)(x - r_2) \) where \( a \) is any nonzero constant.

Solution:

\[
\begin{align*}
\end{align*}
\]

How did you find the activity?

Explain the procedure you have done to determine the equation of the quadratic function.

B. Find the equation of the quadratic function whose zeros are \( 2 \pm \sqrt{3} \).

a. Were you able to get the equation of the quadratic function?

b. If no, what difficulties did you encounter?

c. If yes, how did you manipulate the rational expression to obtain the quadratic function? Explain.

d. What is the equation of the quadratic function?

Study the mathematical concepts below to have a clearer picture on how to get the equation of a quadratic function from its zeros.

If \( r_1 \) and \( r_2 \) are the zeros of a quadratic function then \( f(x) = a(x - r_1)(x - r_2) \) where \( a \) is a nonzero constant that can be determined from other point on the graph.

Also, you can use the sum and product of the zeros to find the equation of the quadratic function. (See the illustrative example in Module 1, lesson 4)

164
Example 1

Find an equation of a quadratic function whose zeros are -3 and 2.

Solution:
Since the zeros are $r_1 = -3$ and $r_2 = 2$, then
\[ f(x) = a(x - r_1)(x - r_2) \]
\[ f(x) = a(x - (-3))(x - 2) \]
\[ f(x) = a(x + 3)(x - 2) \]
\[ f(x) = a(x^2 + x - 6) \] where $a$ is any nonzero constant.

Example 2

Find an equation of a quadratic function with zeros $\frac{3 \pm \sqrt{2}}{3}$.

Solution:
A quadratic expression with irrational roots cannot be written as a product of linear factors with rational coefficients. In this case, we can use another method. Since the zeros are $\frac{3 \pm \sqrt{2}}{3}$ then,
\[ x = \frac{3 \pm \sqrt{2}}{3} \]
\[ 3x = 3 \pm \sqrt{2} \]
\[ 3x - 3 = \pm \sqrt{2} \]

Square both sides of the equation and simplify. We get
\[ 9x^2 - 18x + 9 = 2 \]
\[ 9x^2 - 18x + 7 = 0 \]

Thus, an equation of a quadratic function is $f(x) = 9x^2 - 18x + 7$.

You learned from the previous activities the methods of finding the zeros of a quadratic function. You also have an initial knowledge of deriving the equation of a quadratic function from tables of values, graphs, or zeros of the function. The mathematical concepts that you learned in this section will help you perform the activities in the next section.

What to PROCESS

Your goal in this section is to apply the concepts you have learned in finding the zeros of the quadratic function and deriving the equation of a quadratic function. You will be dealing with some activities and problems to have mastery of skills needed to perform some tasks ahead.
Activity 6: Match the Zeros!

Matching Type. Each quadratic function has a corresponding letter. Similarly, each box with the zeros of the quadratic function inside has a corresponding blank below. Write the indicated letter of the quadratic function on the corresponding blank below the box containing the zeros of the function to get the hidden message.

Y \( f(x) = 4x^2 - 25 \)  
V \( f(x) = 9x^2 - 16 \)  
G \( f(x) = x^2 + 6x + 9 \)  
U \( f(x) = x^2 - 4x - 21 \)  
S \( f(x) = 6x^2 + 5x - 4 \)  
R \( f(x) = x^2 - 9 \)  
E \( f(x) = x^2 - 5x - 36 \)  
L \( f(x) = x^2 - x - 20 \)  
D \( f(x) = 2x^2 + x - 3 \)  
O \( f(x) = 6x^2 - 7x + 2 \)

\[ \{ -3, -3 \}, \{ \frac{2}{3}, 1 \}, \{ \frac{3}{2}, 1 \} \]  
\[ \{ 5, -4 \}, \{ \frac{2}{3}, \frac{1}{2} \}, \{ \frac{4}{3}, \frac{4}{3} \}, \{ 9, -4 \}, \{ -\frac{4}{3}, \frac{1}{2} \} \]

Activity 7: Derive My Equation!

Work in pairs.

A. Determine the equation of the quadratic function represented by the table of values below.

<table>
<thead>
<tr>
<th></th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>-20</td>
<td>-13</td>
<td>-8</td>
<td>-5</td>
<td>-4</td>
<td>-5</td>
</tr>
</tbody>
</table>

B. The vertex of the parabola is (-3, 5) and it is the minimum point of the graph. If the graph passes though the point (-2, 7), what is the equation of the quadratic function?

C. Observe the pattern below and draw the 4th and 5th figures.

Make a table of values for the number of squares at the bottom and the total number of unit squares.

What is the resulting equation of the function?
What method did you use to obtain the equation of the quadratic function in A? Explain how you obtained your answer.

In B, explain the procedure you used to arrive at your answer. What mathematical concepts did you apply?

Consider C, did you find the correct equation? Explain the method that you used to get the answer.

➤ Activity 8: Rule Me Out!

Work in pairs to perform this activity. Write the equation in the oval and write your explanation on the blank provided. Do this in your notebook.

A. Derive the equation of the quadratic function presented by each of the following graphs below.

1. 

2. 

3. 

[Graph images with oval shapes for writing equations]
Explain briefly the method that you applied in getting the equation.

➤ Activity 9: Name the Translation!
Give the equation of a quadratic function whose graphs are described below.

1. The graph of $f(x) = 3x^2$ shifted 4 units downward
2. The graph of $f(x) = 4x^2$ shifted 2 units to the left
3. The graph of $f(x) = 3x^2$ shifted 5 units upward and 2 units to the right
4. The graph of $f(x) = -10x^2$ shifted 2 units downward and 6 units to the left
5. The graph of $f(x) = 7x^2$ shifted half unit upward and half unit to the left

Describe the method you used to formulate the equations of the quadratic functions above.
Activity 10: Rule My Zeros!

Find one equation for each of the quadratic function given its zeros.

1. 3, 2

2. $-2, \frac{5}{2}$

3. $1 + \sqrt{3}, 1 - \sqrt{3}$

4. $\frac{1 + \sqrt{2}}{3}, \frac{1 - \sqrt{2}}{3}$

5. $\frac{11}{3}, -\frac{11}{3}$

In your reflection note, explain briefly the procedure used to get the equation of the quadratic function given its zero/s.

Activity 11: Dare to Hit Me!

Work in pairs. Solve the problem below.

The path of the golf ball follows a trajectory. It hits the ground 400 meters away from the starting position. It just overshoots a tree which is 20 m high and is 300 m away from the starting point.

From the given information, find the equation determined by the path of the golf ball.

Did you enjoy the activities in this section? Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need to be clarified more?
What to REFLECT and UNDERSTAND

Your goal in this section is to have a deeper understanding on how to derive the equation of the quadratic functions. You can apply the skills you have learned from previous sections to perform the tasks ahead. The activities provided for you in this section will be of great help to deepen your understanding for further application of the concepts.

➤ Activity 12: Connect and Relate!

Work in groups of five members. Perform this mathematical investigation.

Joining any two points on a circle determine a chord. A chord is a line segment that joins any two points on a circle. Investigate the relationship between the number of points \( n \) on the circle and the maximum number of chords \( C \) that can be drawn. Make a table of values and find the equation of the function representing the relationship. How many chords are there if there are 50 points on a circle?

What kind of function is represented by the relationship?

Given the number of points, how can you easily determine the maximum number of chords that can be drawn?

How did you get the pattern or the relationship?
Activity 13: Profit or Loss!

Work in pairs. Analyze the graph below and answer the questions that follow.

![Graph of Profit in the Banana Plantation]

a. Describe the graph.
b. What is the vertex of the graph? What does the vertex represent?
c. How many weeks should the owner of the banana plantation wait before harvesting the bananas to get the maximum profit?
d. What is the equation of the function?

Activity 14: What If Questions!

Work in groups with three members each.

1. Given the zeros $r_1$ and $r_2$ of the following quadratic function, the equation of the quadratic function is $f(x) = a(x - r_1)(x - r_2)$ where $a$ is any nonzero constant. Consider $a = 1$ in any of the situations in this activity.
   a. -2 and 3.
   b. $3 \pm \sqrt{3}$.

   A. What is the equation of the quadratic function?
      a. ____________________
      b. ____________________

   B. If we double the zeros, what is the new equation of the quadratic function?
      a. ____________________
      b. ____________________
Find a pattern on how to determine easily the equation of a quadratic function given this kind of condition.

C. What if the zeros are reciprocal of the zeros of the given function? What is the new equation of the quadratic function?
   a. _____________________
   b. _____________________

Find the pattern.

D. What if you square the zeros? What is the new equation of the quadratic function?
   a. _____________________
   b. _____________________

Find the pattern.

2. Find the equation of the quadratic function whose zeros are
   a. the squares of the zeros of \( f(x) = x^2 - 3x - 5 \).
   b. the reciprocal of the zeros of \( f(x) = x^2 - x - 6 \)
   c. twice the zeros of \( f(x) = 3x^2 - 4x - 5 \)

How did you find the activity?

➤ Activity 15: Principle Pattern Organizer!

Make a summary of what you have learned.

Did you enjoy the activities? I hope that you learned a lot in this section and you are now ready to apply the mathematical concepts you learned in all the activities and discussions from the previous sections.
In this section, you will be given a task wherein you will apply what you have learned in the previous sections. Your performance and output will show evidence of your learning.

What to TRANSFER

➤ Activity 16: Mathematics in Parabolic Bridges!

Look for world famous parabolic bridges and determine the equation of the quadratic functions.

Role: Researchers

Audience: Head of the Mathematics Department and Math teachers

Situation: For the Mathematics monthly culminating activity, your section is tasked to present a simple research paper in Mathematics. Your group is assigned to make a simple research on the world’s famous parabolic bridges and the mathematical equations/functions described by each bridge.

Make a simple research on parabolic bridges and use the data to formulate the equations of quadratic functions pertaining to each bridge.

Product: Simple research paper on world famous parabolic bridges

Standards for Assessment:

You will be graded based on the rubric designed suitable for your task and performance.
What to KNOW

The application of quadratic function can be seen in many different fields like physics, industry, business, and in variety of mathematical problems. In this section, you will explore situations that can be modeled by quadratic functions.

Let us begin this lesson by recalling the properties of quadratic functions and how we can apply them to solve real-life problems.

Activity 1:

Consider this problem.

1. If the perimeter of the rectangle is 100 m, find its dimensions if its area is a maximum.
   a. Complete the table below for the possible dimensions of the rectangle and their corresponding areas. The first column has been completed for you.

<table>
<thead>
<tr>
<th>Width (w)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (l)</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area (A)</td>
<td>225</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

   b. What is the largest area that you obtained?
   c. What are the dimensions of a rectangle with the largest area?
   d. The perimeter $P$ of the given rectangle is 100. Make a mathematical statement for the perimeter of the rectangle.
   e. Simplify the obtained equation and solve for the length $l$ of the rectangle in terms of its width $w$.
   f. Express the area $A$ of a rectangle as a function of its width $w$.
   g. What kind of equation is the result?
   h. Express the function in standard form. What is the vertex?
   i. Graph the data from the table in a showing the relationship between the width and the area.
   j. What have you observed about the vertex of the graph in relation to the dimensions and the largest area?
How did you find the activity? Can you still recall the properties of a quadratic function? Did you use them to solve the given problem?
To better understand how the concepts of the quadratic function can be applied to solve geometry problems, study the illustrative example presented below.

Example
What are the dimensions of the largest rectangular field that can be enclosed by 80 m of fencing wire?

Solution:
Let \( l \) and \( w \) be the length and width of a rectangle. Then, the perimeter \( P \) of a rectangle is \( P = 2l + 2w \). Since \( P = 80 \) m, thus,
\[
2l + 2w = 80
\]
\[
l + w = 40
\]
Expressing the length as a function of \( w \)
\[
l = 40 - w
\]
Substituting in the formula for the area \( A \) of a rectangle
\[
A(w) = lw
\]
\[
A(w) = w ( 40 - w )
\]
\[
A(w) = -w^2 + 40w
\]
By completing the square,
\[
A(w) = -(w - 20)^2 + 400
\]
The vertex of the graph of the function \( A(w) \) is \((20, 400)\). This point indicates a maximum value of 400 for \( A(w) \) that occurs when \( w = 20 \). Thus, the maximum area is 400 m\(^2\) when the width is 20 m. If the width is 20 m, then the length is \((40 - 20)\) m or 20 m also. The field with maximum area is a square.

➤ Activity 2: Catch Me When I Fall!

Work in groups with three members each. Do the following activity.
Problem: The height \( H \) of the ball thrown into the air with an initial velocity of 9.8 m/s from a height of 2 m above the ground is given by the equation \( H(t) = -4.9t^2 + 9.8t + 2 \), where \( t \) is the time in seconds that the ball has been in the air.

a. What maximum height did the object reach?
b. How long will it take the ball to reach the maximum height?
c. After how many seconds is the ball at a height of 4 m?
Guide questions:
1. What kind of function is the equation \( H(t) = -4.9t^2 + 9.8t + 2 \)?
2. Transform the equation into the standard form.
3. What is the vertex?
4. What is the maximum height reached by the ball?
5. How long will it take the ball to reach the maximum height?
6. If the height of the ball is 4 m, what is the resulting equation?
7. Find the value of \( t \) to determine the time it takes the ball to reach 4 m.

How did you find the preceding activity? The previous activity allowed you to recall your understanding of the properties of a quadratic function and gave you an opportunity to solve real life-related problems that deal with quadratic functions.
The illustrative example below is intended for you to better understand the key ideas necessary to solve real-life problems involving quadratic functions.

Free falling objects can be modeled by a quadratic function \( h(t) = -4.9t^2 + V_0t + h_0 \), where \( h(t) \) is the height of an object at \( t \) seconds, when it is thrown with an initial velocity of \( V_0 \) m/s and an initial height of \( h_0 \) meters. If the units are in feet, then the function is \( h(t) = -16t^2 + V_0t + h_0 \).

Illustrative example
From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height \( S(t) \) of the ball above the building after \( t \) seconds is given by the function \( S(t) = 80t - 16t^2 \).

1. What maximum height will the object reach?
2. How long will it take the object to reach the maximum height?
3. Find the time at which the object is on the ground.

Solution:
1. The maximum height reached by the object is the ordinate of the vertex of the parabola of the function \( S(t) = 80t - 16t^2 \). By transforming this equation into the completed square form we have,
   \[
   S(t) = 80t - 16t^2 \\
   S(t) = -16t^2 + 80t \\
   S(t) = -16\left(t^2 - 5t\right) \\
   S(t) = -16\left(t^2 - 5t + \frac{25}{4}\right) + 100 \\
   S(t) = -16\left(t - \frac{5}{2}\right)^2 + 100
   \]
The vertex is \( \left( \frac{5}{2}, 100 \right) \). Thus the maximum height reached by the object is 100 ft from the top of the building. This is 196 ft from the ground.

2. The time for an object to reach the maximum height is the abscissa of the vertex of the parabola or the value of \( h \).

\[
S(t) = 80t - 16t^2
\]

\[
S(t) = -16 \left( t - \frac{5}{2} \right)^2 + 100
\]

Since the value of \( h \) is \( \frac{5}{2} \) or 2.5, then the object is at its maximum height after 2.5 seconds.

3. To find the time it will take the object to hit the ground, let \( S(t) = -96 \), since the height of the building is 96 ft. The problem requires us to solve for \( t \).

\[
h(t) = 80t - 16t^2
\]

\[
-96 = 80t - 16t^2
\]

\[
16t^2 - 80t + 96 = 0
\]

\[
t^2 - 5t - 6 = 0
\]

\[
(t - 6)(t + 1) = 0
\]

\[
t = 6 \text{ or } t = -1
\]

Thus, it will take 6 seconds before the object hits the ground.

➤ Activity 3: Harvesting Time!

Solve the problem by following the given steps.

**Problem:** Marvin has a mango plantation. If he picks the mangoes now, he will get 40 small crates and make a profit of Php 100 per crate. For every week that he delays picking, his harvest increases by 5 crates. But the selling price decreases by Php 10 per crate. When should Marvin harvest his mangoes for him to have the maximum profit?

a. Complete the following table of values.

<table>
<thead>
<tr>
<th>No. of weeks of waiting (w)</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of crates</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Profit per crate (P)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Total profit (T)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Plot the points and draw the graph of the function.

c. How did you determine the total profit?

d. Express the profit \( P \) as a function of the number of weeks of waiting.
e. Based on the table of values and graph, how many weeks should Marvin wait before picking the mangoes to get the maximum profit?

This problem is adapted from PASMEP Teaching Resource Materials, Volume II.

How did you find the activity? A quadratic function can be applied in business/industry to determine the maximum profit, the break-even situation and the like.

Suppose \( x \) denotes the number of units a company plans to produce or sell. The revenue function \( R(x) \) is defined as \( R(x) = \text{(price per unit)} \times \text{(number of units produced or sold)} \).

Study the example below.

**Illustrative example**

**Problem:** A garments store sells about 40 t-shirts per week at a price of Php 100 each. For each Php 10 decrease in price, the sales lady found out that 5 more t-shirts per week were sold.

Write a quadratic function in standard form that models the revenue from t-shirt sales.

What price produces the maximum revenue?

**Solution:**

Let \( x \) be the number of additional number of t-shirts sold.

You know that Revenue \( R(x) = \text{(price per unit)} \times \text{(number of units produced or sold)} \).

Therefore, Revenue \( R(x) = \text{(Number of t-shirts sold)} \times \text{(Price per t-shirt)} \)

Revenue \( R(x) = (40 + 5x)(100 – 10x) \)

\( R(x) = -50x^2 + 100x + 4000 \)

If we transform the function into the form \( y = a(x – h)^2 + k \)

\( R(x) = -50(x – 1)^2 + 4050 \)

The vertex is \((1, 4050)\).

Thus, the maximum revenue is Php 4,050.

The price of the t-shirt to produce maximum revenue can be determined by

\( P(x) = 100 – 10x \)

\( P(x) = 100 – 10(1) = 90 \)

Thus, Php 90 is the price of the t-shirt that produces maximum revenue.

**What to PROCESS**

Your goal in this section is to extend your understanding and skill in the use of quadratic function to solve real-life problems.
Activity 4: Hit the Mark!

Analyze and solve these problems.

Problem A.
A company of cellular phones can sell 200 units per month at Php 2,000 each. Then they found out that they can sell 50 more cell phone units every month for each Php 100 decrease in price.

a. How much is the sales amount if cell phone units are priced at Php 2,000 each?
b. How much would be their sales if they sell each cell phone unit at Php 1,600?
c. Write an equation for the revenue function.
d. What price per cell phone unit gives them the maximum monthly sales?
e. How much is the maximum sale?

Problem B.
The ticket to a film showing costs Php 20. At this price, the organizer found out that all the 300 seats are filled. The organizer estimates that if the price is increased, the number of viewers will fall by 50 for every Php 5 increase.

a. What ticket price results in the greatest revenue?
b. At this price, what is the maximum revenue?

c. What properties of a quadratic function did you use to come up with the correct solution to problem A? problem B?

Activity 5: Equal Border!

Work in pairs and perform this activity.

A photograph is 16 inches wide and 9 inches long and is surrounded by a frame of uniform width \( x \). If the area of the frame is 84 square inches, find the uniform width of the frame.

a. Make an illustration of the described photograph.
b. What is the area of the picture?
c. If the width of the frame is \( x \) inches, what is the length and width of the photograph and frame?
d. What is the area of the photograph and frame?
e. Given the area of the frame which is 84 square inches, formulate the relationship among three areas and simplify.
f. What kind of equation is formed?
g. How can you solve the value of \( x \)?
h. How did you find the activity? What characteristics of quadratic functions did you apply to solve the previous problem?
**Activity 6: Try This!**

With your partner, solve this problem. Show your solution.

A. An object is thrown vertically upward with a velocity of 96 m/sec. The distance \( S(t) \) above the ground after \( t \) seconds is given by the formula \( S(t) = 96t - 5t^2 \).
   a. How high will it be at the end of 3 seconds?
   b. How much time will it take the object to be 172 m above the ground?
   c. How long will it take the object to reach the ground?

B. Suppose there are 20 persons in a birthday party. How many handshakes are there altogether if everyone shakes hands with each other?
   a. Make a table of values for the number of persons and the number of handshakes.
   b. What is the equation of the function?
   c. How did you get the equation?
   d. If there are 100 persons, how many handshakes are there given the same condition?

In problem A, what mathematical concepts did you apply to solve the problem?

If the object reaches the ground, what does it imply?

In problem B, what are the steps you followed to arrive at your final answer?

Did you find any pattern to answer the question in d?

**Did you enjoy the activities in this section? Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?**

**What to REFLECT and UNDERSTAND**

Your goal in this section is to have a deeper understanding on how to solve problems involving quadratic functions. The activities provided for you in this section will be of great help to practice the key ideas developed throughout the lesson and to stimulate your synthesis of the key principles and techniques in solving problems on quadratic functions.

**Activity 7: Geometry and Number!**

Solve the problems. Show your solution.

1. What are the dimensions of the largest rectangular field that can be enclosed with 60 m of wire?
2. Find the maximum rectangular area that can be enclosed by a fence that is 364 meters long.
3. Find two numbers whose sum is 36 and whose product is a maximum.
4. The sum of two numbers is 28. Find the two numbers such that the sum of their squares is a minimum?
5. Marlon wants to fence a rectangular area that has one side bordered by an irrigation. If he has 80 m of fencing materials, what are the dimensions and the maximum area he can enclose?

6. The length of a rectangular field is 8 m longer than its width If the area is 2900 m², find the dimensions of the lot.

7. The sum of two numbers is 24. Find the numbers if their product is to be a maximum.

➤ **Activity 8: It’s High Time!**

Work in group with 5 members each. Show your solutions to the problems.

1. A ball is launched upward at 14 m/s from a platform 30 m high.
   a. Find the maximum height the ball reaches.
   b. How long will it take the ball to reach the maximum height?
   c. How long will it take the ball to reach the ground?

2. On top of a hill, a rocket is launched from a distance 80 feet above a lake. The rocket will fall into the lake after its engine burns out. The rocket’s height \( h \), in feet above the surface of the lake is given by the equation \( h = -16t^2 + 64t + 80 \), where \( t \) is time in seconds. What is the maximum height reached by the rocket?

3. A ball is launched upward at 48 ft./s from a platform 100 ft. high. Find the maximum height the ball reaches and how long it will take to get there.

4. An object is fired vertically from the top of a tower. The tower is 60.96 ft. high. The height of the object above the ground \( t \) seconds after firing is given by the formula \( h(t) = -16t^2 + 80t + 200 \). What is the maximum height reached by the object? How long after firing does it reach the maximum height?

5. The height in meters of a projectile after \( t \) seconds is given by \( h(t) = 160t - 80t^2 \). Find the maximum height that can be reached by the projectile.

6. Suppose a basketball is thrown 8 ft. from the ground. If the ball reaches the 10 m basket at 2.5 seconds, what is the initial velocity of the basketball?

➤ **Activity 9: Reach the Target!**

Work in pairs to solve the problems below. Show your solution.

1. A store sells lecture notes, and the monthly revenue, \( R \) of this store can be modelled by the function \( R(x) = 3000 + 500x - 100x^2 \), where \( x \) is the peso increase over Php 4. What is the maximum revenue?

2. A convention hall has a seating capacity of 2000. When the ticket price in the concert is Php 160, attendance is 500. For each Php 20 decrease in price, attendance increases by 100.
   a. Write the revenue, \( R \) of the theater as a function of concert ticket price \( x \).
   b. What ticket price will yield maximum revenue?
   c. What is the maximum revenue?
3. A smart company has 500 customers paying Php 600 each month. If each Php 30 decrease in price attracts 120 additional customers, find the approximate price that yields maximum revenue?

➤ **Activity 10: Angles Count!**

Work in groups of 5 members each. Perform this mathematical investigation.

**Problem:** An angle is the union of two non-collinear rays. If there are 100 non-collinear rays, how many angles are there?

![diagram of angles with 2, 3, and 4 rays]

a. What kind of function is represented by the relationship?
b. Given the number of rays, how can you determine the number of angles?
c. How did you get the pattern or the relationship?

In working on problems and exploration in this section, you studied the key ideas and principles to solve problems involving quadratic functions. These concepts will be used in the next activity which will require you to illustrate a real-life application of a quadratic function.

**What to TRANSFER**

In this section, you will be given a task wherein you will apply what you have learned in the previous sections. Your performance and output in the activity must show evidence of your learning.

➤ **Activity 11: Fund Raising Project!**

**Goal:** Apply quadratic concepts to plan and organize a fund raising activity

**Role:** Organizers of the Event

**Situation:** The Mathematics Club plan to sponsor a film viewing on the last Friday of the Mathematics month. The primary goal for this film viewing is to raise funds for their Math Park Project and of course to enhance the interest of the students in Mathematics.
To ensure that the film viewing activity will not lose money, careful planning is needed to guarantee a profit for the project. As officers of the club, your group is tasked to make a plan for the event. Ms. De Guzman advised you to consider the following variables in making the plan.

a. Factors affecting the number of tickets sold
b. Expenses that will reduce profit from ticket sales such as:
   - promoting expenses
   - operating expenses
c. How will the expenses depend on the number of people who buy tickets and attend?
d. Predicted income and ticket price
e. Maximum income and ticket price
f. Maximum participation regardless of the profit
g. What is the ticket price for which the income is equal to the expenses?

Make a proposed plan for the fund raising activity showing the relationship of the related variables and the predicted income, price, maximum profit, maximum participation, and also the break-even point.

**Audience:** Math Club Advisers, Department Head-Mathematics, Mathematics teachers

**Product:** Proposed plan for the fund raising activity (Film showing)

**Standard:** Product/Performance will be assessed using a rubric.

**Summary/Synthesis/Feedback**

This module was about concepts of quadratic functions. In this module, you were encouraged to discover by yourself the properties and characteristics of quadratic functions. The knowledge and skills gained in this module help you solve real-life problems involving quadratic functions which would lead you to perform practical tasks. Moreover, concepts you learned in this module allow you to formulate real-life problems and solve them in a variety of ways.
Glossary of Terms

axis of symmetry – the vertical line through the vertex that divides the parabola into two equal parts

direction of opening of a parabola – can be determined from the value of a in \( f(x) = ax^2 + bx + c \). If \( a > 0 \), the parabola opens upward; if \( a < 0 \), the parabola opens downward.

domain of a quadratic function – the set of all possible values of \( x \). Thus, the domain is the set of all real numbers.

maximum value – the maximum value of \( f(x) = ax^2 + bx + c \) where \( a < 0 \), is the \( y \)-coordinate of the vertex.

minimum value – the minimum value of \( f(x) = ax^2 + bx + c \) where \( a > 0 \), is the \( y \)-coordinate of the vertex.

parabola – the graph of a quadratic function.

quadratic function - a second-degree function of the form \( f(x) = ax^2 + bx + c \), where \( a \), \( b \), and \( c \) are real numbers and \( a \neq 0 \). This is a function which describes a polynomial of degree 2.

range of quadratic function – consists of all \( y \) greater than or equal to the \( y \)-coordinate of the vertex if the parabola opens upward.

– consists of all \( y \) less than or equal to the \( y \)-coordinate of the vertex if the parabola opens downward

vertex – the turning point of the parabola or the lowest or highest point of the parabola. If the quadratic function is expressed in standard form \( y = a(x - h)^2 + k \), the vertex is the point \((h, k)\).

zeros of a quadratic function – the values of \( x \) when \( y \) equals 0. The real zeros are the \( x \)-intercepts of the function’s graph.

References

Basic Education Curriculum (2002)


Catao, E. et al. PASMEP Teaching Resource Materials, Volume II

Cramer, K., (2001) Using Models to Build Middle-Grade Students’ Understanding of Functions. Mathematics Teaching in the Middle School. 6 (5),


INTEL, Assessment in the 21st Century Classroom E Learning Resources.


Weblinks

Website links for Learning Activities

7. http://www.youtube.com/watch?v=BYMd-7Ng9Y8
15. http://www.youtube.com/watch?v=5bKch8vitu0
Website links for Images